Notes on Life Insurance,

ADDRESSED TO

EDUCATED, INTELLIGENT MEN,

WHO ARE INTERESTED IN THIS SUBJECT, BUT NOT THOROUGHLY ACQUAINTED WITH THE THEORY UPON WHICH LIFE INSURANCE CALCULATIONS ARE BASED, AND THE PRINCIPLES UPON WHICH THE BUSINESS IS FOUNDED.

BY GUSTAVUS W. SMITH.

ATLANTA, GEORGIA, 1869.

NOTES ON LIFE INSURANCE.

IN TWO PARTS.

FIRST:

THEORY OF LIFE INSURANCE.

NET VALUE OF THE RISK ON ONE DOLLAR FOR ONE YEAR.—NET SINGLE PREMIUM.—NET ANNUAL PREMIUM.—TRUST-FUND ON DEPOSIT, OR RESERVE.—NET COST OF INSURANCE.—VALUATION OF POLICIES.—FORMULAS AND TABLES USED IN MAKING NET CALCULATIONS.

SECOND:

PRACTICAL LIFE INSURANCE.

LOADING.—EXPENSES.—SURPLUS.—ADDITIONS TO POLICIES.—LOANS IN PART PAYMENT OF PREMIUMS, AND GENERAL COMMENTS.

"The rate of PREMIUM which MUST BE charged, in order to carry out an Insurance contract, is the problem which stands at the threshold of Life Assurance."—Dr. FARR.

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"In the absence of a popular treatise in which the science of Life Insurance is faithfully and thoroughly interpreted, these annual attempts to throw some light, both on the theory and practice, may be of service."—*Elizur Wright*, 1865.

"The wonderful growth of Life Insurance in this country, since it has been explained and popularized by such essays as those of Professor Wright, seems to prove that, like all good things, it prospers in Light rather than in Darkness."
D. Parks Faekler, 1868.

"Does the system itself rest on principles and laws so certain and stable as to justify a reasonable conviction that, if the system is fairly and honestly administered, the bread that is cast on its waters will be surely found, though after many days?"—*John E. Sanford*, 1868.

INTRODUCTION.

The following notes are not addressed to Officers, Directors, or Agents of Life Insurance Companies; because, in the absence of positive proof to the contrary, it is reasonable to assume that these persons understand the theory and principles of the business they have undertaken to control. In case some of them are not thoroughly informed already, they can no doubt readily procure information from experienced, trained "Experts," Life Insurance Actuaries.

It is far from my purpose, however, to intimate a desire that Life Insurance Officers and Agents shall not read and study these "Notes" if they like. I only disclaim any intention to attempt to guide or teach those now in the business. If an Officer, or Agent of a Life Insurance Company, feels there is something connected with the business he does not clearly understand, and it is inconvenient for the Actuary, or consulting Actuary of his Company, to explain it to him, he may, perhaps, find some of the principles for which he is searching, explained in the following Notes.

It has been well said: "All men should study their own profession, not only with a view to its own peculiar interests, but also as a part of the general mechanism of the world." In other words, *it is essential that every man should understand the business in which he is engaged.* Life Insurance forms no proper exception to this general rule.

In the summer of 1869, I was requested to become a State Agent for a Life Insurance Company. I determined not to accept the offer, until I could, at least, satisfy myself that there was no "mystery in the art" so deeply hidden in "abstruse science" that I could not comprehend it. This led me to an examination of the published official reports of the Insurance Commissioner of Massachusetts, and of the Superintendent of the Insurance Department of the State of New York. The first subject to which my attention was directed, was an essay on the "Contribution Plan of apportioning dividends." The "algebra" of this business was found to be of the simplest elementary character; but the underlying meaning and sense of the thing, was not so clear at first sight. Although the "algebra" and the reasoning upon the "reserve," "net annual premium," and "cost of insurance," were mere *a*, *b*, *c*, and followed as a matter of course, provided the nature and amount of the quantities used were understood, I am bound to say that I knew nothing whatever of the "Contribution Plan" after reading the essay; simply because I knew nothing of the quantities referred to therein, viz: the "reserve," the "net annual premium," and the "cost of insurance." This led me to commence further back. I finally started at the beginning, viz: "the amount that will, if placed at a given rate of interest, produce one dollar certain at the end of the year, when the interest for one year is added to the principal," and then endeavored to follow, step by step, to the end. With the guides I had, and the "lights" before me, I experienced some difficulty in keeping upon the right road; and when, at times, I wandered from it, there was trouble in finding it again.

If by writing and publishing these "Notes on Life Insurance," I may be instrumental in bringing this important subject more clearly before the intelligent, educated business men of the country, thus enlarging the number of the initiated, and bringing new minds into the discussion, I will be more than compensated for the trouble with which I met in trying to follow the "lights" and "signs" set up along the road I traveled when searching out the "mysteries of the art."

I have persistently endeavored to convince the general reader that the "reserve" of which we hear so much, is not "CASH CAPITAL;" nor is a large "reserve" any evidence of *superabundant riches*, or strength, on the part of the Life Insurance Company; in *excess* of the means, necessary to enable the Company to comply with its obligations, and pay its policies at maturity. The fund generally designated by Life Insurance writers as "reserve," is an accrued liability—a *debt*. If a Life Insurance Company desires to give a clear and candid exposition of its affairs, it will accurately compute this "reserve," acknowledge it to be a debt, make it prominent in the statement of its liabilities, and then show, if it can, that the Company has *bona fide* assets to meet *all* its liabilities.

Without further preamble, the general reader is respectfully referred to the following pages, in which, it is believed, he will find the "key" that will enable him, without great difficulty, to investigate and understand the principles upon which the THEORY of Life Insurance is founded, and judge correctly of the PRACTICAL working of this business. The arithmetical examples are introduced solely for illustration, and the results are not to be taken as bald, isolated facts. If errors have been made in the calculations, they can be easily detected and corrected by a good computer; because the principles and the data upon which all the computations are made are fully explained before the arithmetical illustrations are given.

NOTES ON LIFE INSURANCE.

Life Insurance will bear the closest scrutiny. *It needs it.* There is no *magic* in the art, and no *mystery* ought to exist in reference to a subject of such magnitude and serious importance. There are now more than $2,000,000,000 insured upon lives in this country; and the welfare of more than 600,000 families is largely dependent, in case of the death of their natural protectors, upon the prompt payment of the amounts insured.

It would seem that the enormous sum of money involved, as well as the nature of the obligations incurred by these companies, ought to attract the attention of intelligent, business men; not for the purpose of making money; not necessarily for the purpose of taking out policies upon their own lives; but in order to comprehend the nature of this, comparatively new, and already gigantic, element in business—recently introduced by modern civilization—and to judge correctly of the various interests that are directly and indirectly affected thereby.

Life Insurance is rapidly increasing, and must produce either great good, or great evil. It is essential that its peculiarities be clearly understood by those directly concerned; and all intelligent men in the country will readily understand that $2,000,000,000 in any one business, is a sum, which, once jeopardized, might injuriously affect all other values.

A discussion of the general principles of Life Insurance, and the relative merits of the various systems upon which it is conducted, is certainly admissible; and it is hoped the following Notes may be of some interest to those who are not already conversant with the subject.

ABBREVIATIONS.

Mathematical notation will not be of material assistance to the general reader in forming definite and accurate ideas of the principles and quantities that are used in Life Insurance calculations. But, after correct ideas on these points are once clearly formed in the mind of the reader, it will be *convenient*, for both writer and reader, to represent certain quantities by symbols, instead of constantly repeating the whole of the words needed, in the first place, to give a definite idea of the nature and the manner of determining the quantity in question.

In the following pages, for instance, the method of calculating the amount of money that will, if paid in hand, at any named age, be just sufficient to insure one dollar for whole life, will be dwelt on at considerable length. Afterward, this amount will be called "NET SINGLE PREMIUM," and will, for any age x, be represented by the symbol sP_x, and the general reader will, by this symbol, be enabled to recall a definite idea of the process for obtaining this net single premium, and keep in mind an exact idea of what this premium is, as well by seeing the symbol sP_x, as by having the whole process again fully elaborated by the repetition of half a dozen paragraphs.

The amount that will, if paid annually, at the beginning of the year, insure one dollar to be paid to the heirs of the insured at the end of the year in which he may die, is also dwelt upon at length. It is called the "NET ANNUAL PREMIUM," and for any age x, is represented by aP_x.

The money value in hand, at any age x, of a future life series of annual payments of one dollar each, and the method used for obtaining this value, is fully explained hereafter. The first payment, of one dollar, is to be made in hand, and one at the commencement of each following year: *provided*, the person is alive to make the payment; and the present value at age x, of a future whole-life series of annual premiums of one dollar each, is represented by $\cdot A_x$.

There is, in Life Insurance, a peculiar and very important element, which is, by writers on this subject, called by various names. It is often styled the "reserve," or "reserve for reinsurance." Sometimes it is called "net premium reserve," at others "net value," or "true value." It is really an amount of money belonging to the policy-holder, and is held by the Company in trust. This "TRUST FUND DEPOSIT" is, in the following Notes, represented by the letters (T. F. D.), and the amount that should be in deposit at the end of n years from the date of a policy, taken out at the age x, is represented by (T. F. D.)$x+n$. It will be seen hereafter that if a Life Insurance Company has not on hand the requisite amount to make up this trust fund on deposit, and if this amount is not placed to the credit of the respective policies, and regularly increased by net or table interest, compounded yearly, the Company will be unable to pay its policies at maturity.

It should be constantly borne in mind that these abbreviations mean, in every case, "A SUM OF MONEY," the amount or value of which is readily susceptible of direct and easy, but sometimes tedious, arithmetical computation.

(T. F. D.)$x+n$ may, to some readers, be rather a formidable looking symbol; but when they once understand that it is "*a sum of money*," intended for a very special and important purpose, and know exactly how to compute its precise arithmetical amount, and what it is used for, and to whom it of right belongs, and that T stands for *Trust;* F for *Fund;* and D for *Deposit;* that x stands for the age of the policy-holder at the time he first took out his policy, and n for the number of years the policy has been in force, it is believed that the intelligent reader will not find anything very abstruse in (T. F. D.)$x+n$, or in any other of the abbreviations used in these "NOTES."

PART FIRST.

THEORY OF LIFE INSURANCE.

The law of duration of human life, when applied to large numbers of mankind, has been very accurately determined. The perfect uncertainty that a particular individual will survive any definite period, is equaled by the certainty that out of, say 100,000 persons, living at a particular age, a given, and well ascertained number of these, will die in each year, and a certain number will be living at the end of each year. It is the province of Life Insurance, in a pecuniary sense, to enable the insured to replace the uncertainty of his own life, by the certainty of the law governing the general duration of human life, and thus avert the disaster that might befall his family in case of his early death.

The insured, upon paying an annual premium which will cover the risk upon his individual life, enables the Company to pay, with certainty, to his heirs, a stipulated sum whenever he may die. It is proposed to discuss the manner in which this is done by Life Insurance Companies.

Life Insurance calculations are, as a general rule, made on the supposition that the amount insured is one dollar; and having obtained the premium that will insure one dollar, it is an easy matter to determine the premium for a similar kind of policy for any other sum.

In order to determine what amount will be sufficient to insure one dollar, to be paid to the heirs of a person at the end of one year from the date of the payment of the premium—provided the insured dies during the year—we must have at hand a Table of Mortality, and a safe rate of interest must be fixed upon.

The Table of Mortality used in these Notes is the Actuaries', and the rate of interest assumed is four per cent.; but, for the present, we may leave the rate of interest undecided and call it r. We will thereby deduce general rules for the calculations, and can afterwards assign to r any particular value that may be decided upon. The first question to be settled is: what is the amount of money that will, if invested at a rate of interest r, produce one dollar certain at the end of one year? That is to say, the principal and the interest together must amount to one dollar at the end of one year.

THE AMOUNT THAT WILL PRODUCE ONE DOLLAR IN ONE YEAR.

Suppose we represent this amount by v. Then r times v, divided by 100, will represent the interest on v, at the rate r for one year—that is, $\frac{rv}{100}$ is the interest. Add this to the principal, which is v, and the two together must, from the condition imposed, be equal to one dollar. Therefore, we have the equation $v+\frac{rv}{100}=\$1$. Multiply both members of this equation by 100, in order to clear it of fractions, and we have $100v+rv=100$, or, $v(100+r)=100$; hence $v=\frac{100}{100+r}$. We see from this last equation, that the amount of money v, which will, at any rate of interest r, produce one dollar in one year, is obtained by dividing 100 by 100 plus the rate of interest. Give to r any assigned value; substitute for r in the above equation the value thus assigned, and the problem becomes one of very simple arithmetic. For instance, suppose the rate of interest is four per cent., then r is equal to four, and the second member of the equation becomes $\frac{100}{100+4}$, which is the same thing as $\frac{100}{104}$. By performing the division indicated, we have $v=$ \$0.961538; and this is the amount of money which will, at four per cent., produce one dollar in one year. By giving to r any other assigned value, we can, in a manner entirely similar, find the amount which will, at this newly assigned rate of interest, produce one dollar at the end of one year.

THE AMOUNT THAT WILL INSURE ONE DOLLAR FOR ONE YEAR.

But let us now suppose that, instead of having to pay the one dollar, *certain*, at the end of one year, it had been agreed that the person, or company, was to pay the dollar at the end of one year only on condition that the insured died during the year.

If it can be determined accurately what chance, or probability, there is that the insured may die during the year, we have only to multiply the present value of the one dollar, to be paid *certain* at the end of one year, by the fraction which expresses this chance, or probability, in order to determine what it is now worth to insure one dollar, to be paid to the heirs of the insured, in case he dies within one year from the date of the transaction.

The question, whether a person now in good health may die within a year, if applied to a single individual, is to the last degree uncertain; but close observation of accurate statistics, has established the fact, that there is a fixed general law governing the duration of human life, when applied to large numbers of mankind; and it is known, that, of any large number of people living at any particular age, a certain proportion of these will die each year. For instance, suppose we take 100,000 persons living at ten years of age; during the year between age ten and age eleven, 676 of these persons will die, and there will be 99,324 of them living at the age eleven. In the year, between age eleven and age twelve, a given number will die, and in each successive year a given number will die, until the last passes away in death.

The results of statistical observation upon the duration of human life have been carefully tabulated and combined in what are called "Mortality Tables." In these tables are recorded the number living at each age, and the number of these that will die within the following year. Now, suppose that there are a large number of persons of any given age, insured in a particular Company, enough to cause the average mortality amongst the insured to conform to the law of mortality, as expressed in the table. We can, by using the Mortality Table, not only find the chance or probability that a particular individual of those thus insured may die during any year, but in applying this to all the insured, the business would be entirely divested of that element of pure gambling upon chances, upon which it would be founded, if applied to a single individual, or even a small number of individuals. There is nothing more true than that "Life Insurance seeks breadth of basis, and can not be safely cooped within narrow limits."

With an accurate Mortality Table at hand, we could find the number of persons living at the age of the person who desires to have, say one dollar, insured to be paid to his heirs at the end of one year, in case he dies during the year. Of the number living at the beginning of the year, the table gives the number that will die during the year. Now, on the supposition that, of the whole number living at the beginning of the year, each has as good a chance of living, or dying, as another, we obtain the chance or probability that any given one of the number will die during the year by dividing the whole number of deaths during the year by the whole number living at the beginning of the year. This will give the fraction which represents the chance or probability that the insured may die during the year.

The Fraction that Represents the Chance or Probability that the Insured may Die During any Year.

In illustration of the calculation of the chance or probability that a person may die during any given time, let us suppose that out of one hundred persons condemned to be shot on a given day, all are reprieved, for the day, except one, and the one to be shot is to be the one who draws the black ball out of a box containing ninety-nine white balls, and one black one. The chance, before the drawing, of any particular man's getting the black ball, and, therefore, his chance of being shot that day, is one only out of one hundred, or $\frac{1}{100}$ of a certainty. If two men had to die that day, each individual's chance, before the drawing, of getting a black ball, would be twice as great as it was before, for now there are two black balls and only ninety-eight white ones in the box. The chance, in this case, would be $\frac{2}{100}$ of a certainty; for three persons $\frac{3}{100}$, and so on to the limit of one hundred one hundredths, which would make it certain that each man would be shot, because all the one hundred balls were black.

To apply this principle to Life Insurance, and to show "that the amount that will insure one dollar to be paid certain at the end of any year, when multiplied by the fraction which represents the chance or probability that the person will die during the year, gives what it is worth to insure one dollar, to be paid at the end of the year, in case the insured dies during the year." Let us suppose that out of one hundred persons alive at the beginning of the year, it is known that one of them, and one only, will die during the year. The value of one dollar, to be paid certain at the end of one year, is represented by v, which, in the particular case of interest at four per cent., has been shown to be $0.961538. The chance that the person will die during the year, is one out of one hundred, or the one one-hundredth part of a certainty. To make it certain that his heirs will obtain one dollar at the end of the year, he must advance $0.961538 at the beginning of the year. But the one dollar is not to be paid certain; it is to be paid only in case he dies. Suppose the whole one hundred persons are insured; then, since there is but one to die, and but one dollar to be paid, each person will have to give only the one-hundredth part of v in order to make up the whole of v. Therefore, $\frac{1}{100}$, multiplied by v, is what each man would have to pay. In case it is known that two persons out of the one hundred will die, the amount requisite to effect the insurance will be twice as much as before, because two dollars must be paid certain at the end of the year. The chance or probability that each person may die during the year is twice as great as in the first case, and is therefore two out of one hundred; and the amount that each person will have to pay for his insurance is $\frac{2}{100}$ multiplied by v. The other ninety-eight persons will get nothing; they have been insured for one year, and have paid for that insurance. The aggregate of all the payments was $2v$, and this, at four per cent., produced two dollars *certain* at the end of the year, with which to pay the heirs of the two persons that died.

In case it is known that three persons of the one hundred will die during the year, each will have to pay $\frac{3}{100}$ of v; and the chance or probability that any individual of the whole number will die during the year is represented by the fraction $\frac{3}{100}$. In case it is known that all of them will die during the year, the fraction which represents the chance or probability that any particular individual will die, becomes $\frac{100}{100}$, or unity. This represents the certainty; and to insure one dollar to be paid at the end of the year, to the heirs of the insured, in case he dies during the year, each person will now have to pay $\frac{100}{100}$ of v, which is equal to v; that is to say, each must, in this case, pay enough to make, with interest at four per cent. added, the full amount for which he is insured.

To Insure One Dollar for One Year at Age 30.

To return to the Mortality Table: Suppose the person to be thirty years of age. The table shows that out of 100,000 persons living at the age ten, there will be 86,292 living at the age thirty. The number of deaths between age thirty and age thirty-one is 727. Therefore, $\frac{727}{86{,}292}$ is the fraction which represents the chance or probability that the insured will die before he is thirty-one years of age.

The present value of one dollar, to be paid *certain* at the end of one year, has, in the case of interest at four per cent., been found to be equal to \$0.961538. Now, multiply this by the fraction $\frac{727}{86{,}292}$, which represents the chance or probability that the insured will die during the year between age thirty and age thirty-one, and we have $0.00810083. This is *the value of the risk on one dollar*, or what it is worth to insure one dollar to be paid to the heirs of a person at the end of one year, in case he dies during the year, the age of the insured being thirty years at the time he took out the policy. It would require one thousand times as much to insure one thousand dollars as it does to insure one dollar, and half as much to insure half a dollar as it does a whole dollar. We can obtain, by using the Mortality Table, the fraction representing the chance or probability of a person dying during the year at any named age, just as we did in this case for age thirty. We have, therefore, already determined the means for calculating the sum that will insure any given amount to be paid to the heirs of the insured in one year, in case he dies during the year; but we must know the age of the person, the rate of interest must be fixed, and a Mortality Table must be available for use in the calculations.

To Insure Ten Thousand Dollars for One Year at Age Forty.

Suppose, for instance, the age of the person applying for insurance is forty; the Mortality Table used is the Actuaries', and the rate of interest is four per cent.; the insurance to be for one year; the amount of the policy ten thousand dollars, and the insured is a *fair average* of mankind *in health*, and *prospect of longevity.* What amount in hand ought to be paid to insure ten thousand dollars as above, leaving out all consideration of expenses, and having neither gain nor loss represented in the chances of the transaction?

We will first determine the amount that will insure one dollar. We have before found that the present value of one dollar, to be paid *certain* at the end of one year, interest being assumed at four per cent. per annum, is equal to $0.961538. From the Mortality Table we find that out of 100,000 persons living at the age ten, there are 78,653 living at the age forty, and that 815 of these will die during the year between the age forty and the age forty-one. Therefore, $\frac{815}{78,653}$ is the fraction which represents the chance or probability that the insured will die during the year; and this multiplied by \$0.961538, which is the present value of one dollar, to be paid *certain* at the end of one year, will give what it is now worth to insure the one dollar, to be paid in case the insured dies during the year, or \$0.961538 $\times \frac{815}{78,653}$ = \$0.009963428. This multiplied by 10,000, makes \$99.63, plus a fraction of a cent, which is the net single premium that will insure ten thousand dollars, to be paid to the heirs of the person insured at the end of one year: provided he dies during the year; and in case the whole 78,653 persons had been insured, the 815 who died would each have had ten thousand dollars paid to their heirs, and there would have been nothing left at the end of the year's business. Every one of the whole 78,653 had been insured for one year in the sum of ten thousand dollars each; but only the heirs of those who died were respectively entitled to, and were paid, the ten thousand dollars. The aggregate payment of losses by death, for the year, amounted, therefore, to \$8,150,000, and the net annual premiums \$99.63 each; when increased by four per cent. interest, amounted to \$8,149,996.43. The omitted decimals on a single net annual premium amounts to three dollars and some cents, when carried into as many as 78,653 policies.

PRESENT VALUE THAT WILL PRODUCE ONE DOLLAR CERTAIN IN TWO YEARS.

Now the question arises, what will be the present price or value necessary to insure one dollar, to be paid to the heirs of the insured in case he dies during the second year?

We will first find, as before, the present value of the one dollar to be paid *certain;* but in this case it is payable at the end of *two* years. We will designate the value payable *certain* at the end of two years by v'', and call the rate of interest r, as before. Now, if we multiply v'' by the rate of interest r, and divide the product by 100, we will obtain an expression which represents the interest on v'' at the rate r

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for the first year. The interest for the first year is therefore represented by $\frac{rv''}{100}$. Add this interest to the principal v'' and we have $v''+\frac{rv''}{100}$, which is the sum to be placed at interest at the beginning of the second year. This sum $v''+\frac{rv''}{100}$, multiplied by r, and the product divided by 100, will give us $\frac{r}{100}\left(v''+\frac{rv''}{100}\right)$, which is the interest during the second year. The original sum v'', with the interest for the first year and the interest for the second year added to it, is equal to one dollar; therefore we have $v''+\frac{rv''}{100}+\frac{r}{100}\left(v''+\frac{rv''}{100}\right)=\$1$. Multiplying both members of this equation by 10,000, in order to clear it of denominators, we have $10,000\,v''+200rv''+r^2v''=10,000$, or $v''(10,000+200r+r^2)=10,000$. Hence, $v''=\frac{10,000}{10,000+200r+r^2}$. Substituting in this expression, which gives the value of v'', any given or assigned value of r, and it becomes a question of very simple arithmetic to obtain the numerical value of v'', at any assigned rate. Let us suppose the rate of interest is as before, viz: 4 per cent. per annum; that is $r=4$, and we find, $v''=\frac{10,000}{10,000+800+16}$ $=\frac{10,000}{10,816}=\$0.924556$.

Before proceeding further, it is well to note the fact that the algebraic expression for v, in terms of r, viz: $v=\frac{100}{100+r}$ will, when multiplied by itself, or raised to the second power, become $v^2=\frac{10,000}{10,000+200r+r^2}$; and this is the precise expression found above for the value of v''. Therefore, $v''=v^2$; that is to say, the present value of one dollar, payable certain at the end of two years at any rate of interest r, is equal to the present value of one dollar, payable certain at the end of one year, at the same rate of interest, raised to the second power. And so, too, the arithmetical value of v, multiplied by itself, will produce the arithmetical value of v''; that is, \$0.961538 $\times$\$0.961538=\$0.924556.

Notice that v is always less than one dollar; its value is, therefore, a fraction of unity, and multiplying v by itself, the resulting value must be less than v.

Present Value that will Produce One Dollar Certain in n Years.

Calling the present value of one dollar, to be paid certain at the end of three years, v''', and placing this at a rate of interest r, we find in a similar manner an algebraic expression for the value of v'''. Having found this value, an inspection of the algebraic expression will show that v''' is equal to v raised to the third power; the same will be true for the arithmetical values of v and v'''.

In short, it is an algebraic law that the present value of one dollar (computed at any given rate of interest), to be paid *certain* at the end of one year, will, when raised to a power, the exponent of which is n, be equal to the present value of one dollar, to be paid certain at the end of n years, interest being compounded annually.

NET SINGLE PREMIUM.

To Calculate the Net Single Premium at Age x, that will Insure One Dollar for Whole Life.

We are now ready to calculate the net single premium that will insure one dollar, to be paid to the heirs of a person at the close of the year in which he may die; that is, to determine what sum paid in hand will, on the supposition that the Mortality Table is correct, and the assumed rate of interest is always realized and compounded yearly, be the exact equivalent of one dollar insured, to be paid at the end of the year in which the person insured may die.

We have seen how to calculate the present value of one dollar, to be paid certain at the end of one, two, three, or any number n of years. We have seen, too, that the Mortality Table furnishes the data for determining the fraction which expresses the chance or probability of the insured dying in any year. Let us suppose, now, that the person desiring to be insured is thirty years of age. We have previously seen that the value of v is equal to $0.961538; and that the fraction which expresses the chance or probability of the person's dying between the age thirty and the age thirty-one, is $\frac{727}{86,292}$; and that the value of one dollar, to be paid certain at the end of one year, multiplied by the probability of the insured dying during the year, gives for the amount that would insure one dollar for one year, $0.00810083.

Now, calculate in a similar manner the present value that will insure one dollar, to be paid at the end of two years, provided the insured dies during the second year. In this case v becomes v^2. The table gives the number of deaths between age thirty-one and age thirty-two; this number, divided by the whole number living at age thirty, will give the present chance or probability that the insured will die during the second year. The calculation is made in a similar manner for the third year, fourth year, and for every year up to and including age ninety-nine, which is, by the Table of Mortality we are using, the practical limit of human life. All these respective yearly present values are added together, and the sum gives the amount of the net single premium that would just insure one dollar, to be paid to the heirs of a person at the end of the year in which he may die; the age of the person at the time of taking out his policy being, in this case, thirty years.

For any age different from that assumed, the numbers taken from the table will be found opposite to the age, and the fraction representing the chance or probability of the person's dying varies with the age. But v is always the same, for the rate of interest assumed, four per cent., viz: $0.961538 for the first year, and the square of this for the second year; and the present value of one dollar, to be paid certain at the end of n years, is equal to v, raised to a power whose exponent is n.

Having found the net single premium that will insure one dollar for life, any other amount at the same age will be insured by a proportional net single premium. We have, therefore, shown how the net single premium may be calculated that will insure at any age any given amount, to be paid to the heirs of the insured at the end of the year in which he may die. It requires a good deal of plain arithmetic.

In case the insured is not a very old man at the time of taking out his policy, a great deal of ciphering will be necessary in order to make the calculation for determining the net single premium that will insure one dollar for whole life. For instance, at age twenty the calculation must be made for each year separately, from twenty to ninety-nine, inclusive; and these yearly values must all be added together. At age ninety-nine, however, in case a person at that age desired to be insured, the calculation for the net single premium is readily made, and with very little actual ciphering, because ninety-nine is the limit of the duration of human life, according to the Table of Mortality we are now using. We will, therefore, have to calculate for but one year, and the fraction which represents the chance or probability that the insured will die during the year, is equal to one divided by one, or unity.

It follows, therefore, that the amount required at age ninety-nine, to insure one dollar to be paid to the heirs of the insured at the end of the year in which he may die, is equal to the amount in hand that will, at the end of one year, when table or net interest is added, be just equal to one dollar; that is to say, at age ninety-nine, the net single premium that will insure one dollar for whole life is equal to v. From this it is seen, that, in case only the value of the actual risk on one dollar for each successive year is paid, the price for each year increases, until at age ninety-nine, the amount the insured will have to pay at that age is the amount that will, at table interest, produce one dollar certain in one year.

Insuring for each Separate Year, and Insuring by Net Single Premium for Whole Life in Advance.

It is therefore seen, that, in the later years of a long life, if the method of insuring for only one year at a time is followed, the yearly premium will finally become almost equal to the amount of the policy. This plan of paying each year only the value of the risk during that year on one dollar will, in time, become very burdensome to the policy-holder, and the result will be very unsatisfactory to those who live for a long period.

On the other hand, the payment of a net single premium for whole life insurance requires a very large sum in advance. By this single payment the insured places at once in the hands of the Insurance Company an amount which is sufficient, when increased by net interest for the first policy-year, to pay the actual value of the risk on his policy during that year, and leave in the hands of the Company at the end of the first year an amount equal to the net single premium due to the age $x+1$; and so for each successive year to the limit of the Table of Mortality.

To obviate the difficulties inherent in these two methods of insurancc, viz: in one case a constantly increasing premium, in case the insured pays each year only the value of the risk during that year; in the other a very heavy payment made at once in advance, that covers the value of the risk during every year; a system of *equal annual payments* has been devised, so adjusted as to be the precise equivalent in money value of the net single premium: this, on the supposition that the Mortality Table is exact, and that net or table interest is always realized and compounded yearly. By this method of *equal annual payments* the insured, during the earlier years of his policy, pays more than the value of the risk each year. This excess of payment is increased by table interest, compounded, and in the later years of a long life will be just sufficient to counterbalance the *deficiency*, during these years, in the *annual premium.*

NET ANNUAL PREMIUM.

Having seen how to calculate the net single premium that will insure any given amount to be paid to the heirs of a person at the end of the year in which he may die, we will now proceed to show what the value of each one of a series of equal annual premiums must be in order to insure one dollar to be paid to the heirs of a person at the end of the year in which he may die, the first payment being made in advance, and one at the beginning of each succeeding year; provided the person is alive to make the payment.

Value of a Life Series of Annual Premiums of One Dollar Each.

To enable us to do this, we will first find the present value of a life series of annual payments or premiums of one dollar, the first in advance. The first term of this series is unity, or one dollar, because the first payment is in advance. The second term is equal to the present value of one dollar, to be paid certain at the end of one year, multiplied by the fraction which represents the present chance or probability of the person being alive at the end of the year to make the payment. We will suppose that the person is thirty years of age. In the table we find the number living at age thirty is 86,292; and the number living at thirty-one is 85,565. We therefore obtain the fraction representing the chance or probability of the person being alive to make the second payment, by dividing the number living at thirty-one by the number living at thirty, which is $\frac{85{,}565}{86{,}292}$. This multiplied by v, which, as before found, is equal to \$0.961538, will give the present value of the second payment; it is \$0.953437.

To obtain the present value of the third payment, we have the present value of one dollar, to be paid certain at the end of two years, equal to v^2. From the Mortality Table we find the whole number living at the end of two years, and by dividing this by the whole number living at age thirty, we obtain the fraction which represents the present chance or probability that the person will be alive to make the third payment.

Then multiply v^2 by this fraction, and we have the present value of the third payment. In a similar manner the calculations are made for each year up to, and including, ninety-nine years of age. All these respective yearly present values are added together, and their sum gives us the present value of a life series of annual payments of one dollar, the first in advance—the age thirty years. By a process entirely similar, the calculation can be made for any other age. We have, therefore, shown how to calculate the value at any age of a series of annual payments of one dollar—the first payment being made in advance, the others to be paid at the beginning of each succeeding year; *provided* the person is alive to make the payments. It is clear that the present value just found bears the same proportion to an annual premium of one dollar as any other present value or amount in hand must bear to its equivalent annual premium. In other words, if, at any age, it is found that the present value of a life series of annual premiums or payments, each equal to one dollar, is a given sum, then the present value of a similar series of payments or annual premiums, each equal to two dollars, will be twice as much; and the present value of a similar series of annual premiums, of half a dollar each, will be half as much, and in proportion for any other similar series of annual premiums.

If we once find by arithmetical calculation, as above, the present value of a life series of future premiums of one dollar each, at any given age, we have only to multiply this value by the amount of any other given annual premium in order to find the present value at that age of a life series of these premiums.

We represent the present value of a life series of future annual premiums or payments of one dollar, at any age, by A, and A_x represents this value at the age x. Call the net single premium that will insure one dollar for life sP, and let sP_x represent the net single premium that will insure one dollar for life at the age x. Since an amount A_x in hand is the equivalent at the age x of a future life series of annual premiums of one dollar, an amount sP_x in hand will be the equivalent present value at same age of a *proportional* similar series of future annual premiums. In other words, A_x is to sP_x as one dollar, the equivalent annual premium of A_x is to the equivalent annual premium of sP_x. This proportion gives us $\frac{sP_x}{A_x}$ as the general expression for the value of an annual premium; the equivalent in hand for the future series of which is sP_x.

But sP_x in hand will insure one dollar for life at the age x; therefore a life series of annual premiums, each equal to $\frac{sP_x}{A_x}$, will insure one dollar for life at age x. We represent this net annual premium by aP; and aP_x represents the net annual premium at the age x; and we have $aP_x = \frac{sP_x}{A_x}$. That is to say, the net annual premium, aP_x, that will insure one dollar for life at the age x, is equal to the net single premium that will insure one dollar for life at the same age, which is sP_x divided by the value at that age of a future life series of annual premiums of one dollar, which we have agreed to call A_x.

Relation Existing Between sP_x and A_x.

It is well to show here the peculiar relation existing between the value at any age of a future life series of annual premiums of one dollar, and the net single premium that will insure one dollar for life at the same age. This net single premium is obtained by considering each year separately, and first finding the present value or amount that will, at net interest, produce one dollar certain at the end of each year, and then multiply this value by the fraction which represents the present chance or probability that the insured may die in each year, and adding these results together.

Let l represent the number living, then l_x will represent the number living at the age x, and l_x+1 will represent the number living at the age $x+1$, &c., &c. The number of deaths in any year is equal to the number living at the beginning of the year, minus the number living at the end of the year. The number of deaths during the year, between the age x and the age $x+1$, will then be represented by $l_x - l_{x+1}$; and for the year between the age $x+1$ and the age $x+2$, the number of deaths will be represented by $l_{x+1} - l_{x+2}$. With this notation explained, we can, by referring to the method previously indicated by which to obtain the net single premium that will insure one dollar, to be paid to the heirs of a person aged x, at the end of the year in which he may die, at once write out the following equation:

$$sP_x = \frac{v(l_x - l_{x+1})}{l_x} + \frac{v^2(l_{x+1} - l_{x+2})}{l_x} + \frac{v^3(l_{x+2} - l_{x+3})}{l_x} +, \text{\&c.}$$

This series of terms is continued up to age ninety-nine. By separating the positive and negative terms, and placing all the positive terms of the numerators together, and dividing them by their common denominator, and arranging the negative terms in a similar manner, the above equation becomes, by placing v as a common factor of all the positive terms:

$$sP_x = v\frac{l_x + vl_{x+1} + v^2 l_{x+2} +, \text{\&c., carried out to age ninety-nine.}}{l_x}$$

$$-\left(\frac{vl_{x+1} + v^2 l_{x+2} + v^3 l_{x+3} +, \text{\&c., carried out to age ninety-nine.}}{l_x}\right)$$

Now, by using the same notation, and bearing in mind the manner in which the present value of a life series of annual payments of one dollar, the first in advance, was calculated, we are enabled to write (*noticing that the first payment of one dollar is expressed in the first term of the expression by* $\frac{l_x}{l_x}$, *which is equal to unity, and can, therefore, be written to represent the one dollar paid in advance*), the following equation:

$$A_x = \frac{l_x + vl_{x+1} + v^2 l_{x+2} +, \text{\&c., carried out to age ninety-nine.}}{l_x}$$

By comparing this value of A_x with the equation that gives the value of sP_x just above, it is seen that the positive term in the expression for that value is equal to vA_x, and that the negative term would exactly correspond with the value of A_x if unity or $\frac{l_x}{l_x}$ were added to this negative term. Therefore, if the negative term, with unity added to it, is equal to A_x, the negative term as it stands is equal to $A_x - 1$.

By substituting in the equation that gives the value of sP_x, the expression A_x for its equivalent terms, we have—

The Formula Used in Calculating the Net Single Premium.

$$sP_x = vA_x - (A_x - 1) = 1 + (v-1)A_x = 1 - (1-v)A_x.$$

But we have previously seen that $aP_x = \frac{sP_x}{A_x}$. Substituting in this equation, in place of sP_x, its value just obtained in terms of v and A, and we have—

THE FORMULA USED IN CALCULATING THE NET ANNUAL PREMIUM.

$aP_x = \frac{1-(1-v)A_x}{A_x} = \frac{1}{A_x} - (1-v.)$ This is the formula used in calculating the net annual premium that will insure one dollar for life at the age x. If we can obtain the value of A at any age, the net single premium and the net annual premium to insure one dollar for life at that age may be readily obtained from the above equations, which give their respective values in terms of A and v.

THE NUMERICAL VALUE OF A_x.

Referring to the equation that expresses the value of A_x, and multiplying both the numerator and denominator of the fraction, which expresses that value by v^x—which will not change the value of the fraction—we have a new equation for A_x, as follows:

$$A_x = \frac{v^x l_x + v^{x+1} l_{x+1} + v^{x+2} l_{x+2} +, \text{\&c., to, and including, age 99.}}{v^x l_x}$$

Let us now suppose that the age is ninety-nine years, which is the limit of the duration of human life according to the Table of Mortality we are using. The equation would become in this case $A_x = \frac{v^x l_x}{v^x l_x}$ or $A_{99} = \frac{v^{99} l_{99}}{v^{99} l_{99}} = 1$; because this is the last as well as first term of the series; and, placing this value of A_x in the expression for aP_x, which is $\frac{1}{A_x} - (1-v)$, the net annual premium that would insure one dollar to be paid to the heirs of a person aged x at the end of the year in which he may die, it becomes $aP_{99} = \frac{1}{1} - (1-v)$ $= 1 - (1-v) = v$; that is to say, the net annual premium that will insure one dollar to be paid to the heirs of a person aged ninety-nine, at the end of the year in which he may die, is equal to the present value of one dollar, to be paid certain at the end of one year, interest being assumed at the rate r, which, in our calculations, is four per cent. This ought to be so under the assumptions made, because the person, according to the table we are using, must die certain during the ninety-ninth year.

When $x=99$, the equation becomes then $A_{99}=\frac{v^{99}l_{99}}{v^{99}l_{99}}$. Suppose $x=98$, $A_{98}=\frac{v^{98}l_{98}+v^{99}l_{99}}{v^{98}l_{98}}$. Suppose $x=97$, we have $A_{97}=\frac{v^{97}l_{97}+v^{98}l_{98}+v^{99}l_{99}}{v^{97}l_{97}}$; and thus diminishing the age each year by one, it is seen that the numerators not only have for their first term the denominator pertaining to that year, but the second term of the numerator is the denominator of an age one year greater; and the third term of the numerator is the denominator of an age two years greater; and so for other ages. It follows from this reasoning, that if we call the denominator at any age x, D_x, that the numerator will be expressed by $D_x+D_{x+1}+D_{x+2}+D_{x+3}+$ . . . . . . to D_{99}. If we call the sum of all these terms of the numerator at the age x, N_x, we will have the equation $A_x=\frac{N_x}{D_x}$.

These numerators N_x, and denominators, D_x, have been accurately calculated for the different ages, and the results correctly tabulated. The table gives opposite the respective ages, the numerators in a column headed N_x, and the denominators in a column headed D_x. Then, in order to obtain, by using these tables, the present value, at any age, of a life series of annual payments of one dollar, the first in advance (having assumed the Actuaries' Table of Mortality and four per cent. interest per annum, compounded yearly); we look in the table opposite the given age, and in the column headed N_x we find the numerator, and in the column headed D_x we find the denominator; and we thus obtain the fractional expression which is the value sought for. For example, let the age be taken at thirty years: the equation $A_x=\frac{N_x}{D_x}$, becomes for this case $A_{30}=\frac{N_{30}}{D_{30}}$. In the table opposite age thirty we find the numerator in the N_x column; it is 479,951.6; and in the D_x column we find the denominator is 26,605.37. Therefore we have $A_{30}=$

$\frac{479,951.6}{26,605.37}$=\$18.039. In a precisely similar manner we find the arithmetical value of A_x at any other age.

WHAT IS THE NET ANNUAL PREMIUM THAT WILL INSURE ONE THOUSAND DOLLARS FOR LIFE AT THE AGE FORTY-TWO?

The formula is $aP_x=\frac{1}{A_x}-(1-v)$. We must first find the value of A_x at the age forty-two. $A_x=\frac{N_x}{D_x}$, which in this case becomes $A_{42}=\frac{N_{42}}{D_{42}}=\frac{231,671.7}{14,830.58}=15.621$. Therefore the equation is, in this case, $aP_{42}=\frac{1}{15.621}-(1-v)$. But we have previously found v to be equal to \$0.961538; therefore $(1-v)$ =\$0.038462. The equation then is $aP_{42}=\frac{1}{15.621}-0.038462$. By performing the division indicated in the fraction $\frac{1}{15.621}$ we find this fraction to equal 0.0640164. Therefore $aP_{42}=$ 0.0640164—0.038462=\$0.025554. This is the net annual premium that will insure one dollar to be paid to the heirs of a person aged forty-two years, at the end of the year in which he may die. Multiply this by 1,000 and we find the net annual premium that will insure \$1,000 for same age is equal to \$25.55.

In a similar manner the net annual premiums for whole life policies are calculated at any age and for any amount.

TRUST FUND DEPOSIT, OR "RESERVE."

The "Reserve" has been well styled by the highest authority, "The Great Sheet-Anchor of Life Insurance." It is essential that its bearings upon the practical business of Life Insurance be clearly understood by all who have anything to do with this subject.

We have just calculated the net annual premium that will insure one thousand dollars for life at age forty-two. It will be borne in mind that this premium is to be paid at the beginning of each year, provided the person is alive to make the payment. At the end of the first year, or beginning of the second—supposing the insured to be alive—he pays the net annual premium, $25.55, and is insured for another year; but he is now forty-three years old.

What is the net annual premium that will insure one thousand dollars for life at age forty-three?

Making the calculation in a manner entirely similar to that just gone through with for age forty-two, only substituting age forty-three in this for age forty two in that example, and it will be found that a net annual premium of $26.58 is required to insure one thousand dollars for life at age forty-three. Why is it that the man who was insured at age forty-two, and who has been insured one year, and has paid for that insurance, can, at forty-three years of age, be insured by the Company for a less premium than is required to insure a man of the same age, forty-three; but who now takes out a policy for the first time in that Company. Taking for further illustration a still greater age, we find that at age sixty-five the net annual premium that will insure one thousand dollars for life is $74.72; and yet the person who took out his policy at age forty-two, supposing he is still alive, can be safely insured at age sixty-five by the Company for a net annual premium of $25.55.

How is this? Why is it that a man sixty-five years of age can be insured safely by a Company for a net annual premium of $25.55; and another man of the same age, probably in better health, because he has just passed a medical examination, can not be safely insured by the Company for a less net annual premium than $74.72? It is not a full answer to say that one of the persons has been insured since he was forty-two years of age, and, therefore, don't have to pay so much. Nor is it very satisfactory to most intelligent business men to be told, "This is so, because it is thus put down in the tables." There are now largely over half a million of policies on lives in force in the United States, insuring more than two thousand millions of dollars; and it would seem that the amount involved, the nature and character of the obligation incurred, and the immense interests that are directly and indirectly involved in Life Insurance business, as it stands in this country to-day, would attract the close attention of intelligent men.

However well a person may understand how to calculate the net annual premium, unless he also understands the nature and object of the "Reserve," as it is technically called, he knows *nothing* of the real business of Life Insurance; more especially in these days, when "surplus" and "dividends" seem to be all the rage, both with policy-holders and the share-holders of nearly all the stock companies. Even the "*purely*" mutual companies sometimes, if not generally, style their "reserve" "CASH CAPITAL," *when it is, in fact, an accrued liability of the company—a* DEBT.

The net annual premium is calculated to provide against all the probabilities and risks of the insured dying in any year, and of his policy becoming due; and also the risk of his being alive, from year to year, to pay his annual premium. *The net annual premium, at the rate of net interest, assumed to be always realized and compounded yearly, is exactly sufficient to pay its proportion, year by year, of losses that will occur by the death of a certain number of policy-holders, as given by the Mortality Tables used; and at the same time provide for the payment of the policy at the death of the policy-holder.* At the end of each year, after the net annual premium has paid its proportion of the losses by death for the year, there must be in the hands of the Company, on account of, *and to the credit of*, each and every outstanding policy, an amount in money or securely invested funds, that will be in present value, in hand, the equivalent of an annual premium, equal to the difference between the net annual premium the insured paid on taking out his policy and the net annual premium he would now have to pay if he were taking out a new policy at his present advanced age. This amount that must be in the hands of the Company at the end of each year's business, to the credit of the respective policies, is variously styled, by Life Insurance writers, "reserve," "reserve for reinsurance," "net premium reserve," "net value," and "true value." In these remarks it will be called "TRUST FUND DEPOSIT;" understanding that IT IS AN ACCRUED LIABILITY

OR DEBT, NOT "CASH CAPITAL." We now propose to see how it is computed.

Referring to the value of the net annual premium already calculated, that will insure $1,000 for life, at age forty-two, which is $25 55; and the net annual premium to insure the same amount, at age forty-three, which, as seen above, is $26.58, we find the difference between these two annual premiums to be $1.03.

We have previously shown that the present value of a life series of annual premiums of one dollar, at any age x, is equal to A_x. At age forty-three A_x becomes $A_{43}=\frac{N_{43}}{D_{43}}$ $=\frac{216,841.2}{14,104.81}=15.373564$.

Now, the question simply is this: If a life series of annual premiums of one dollar, at the age forty-three, is equal to a present value of $15.373564, what is the present value at same age of a life series of annual payments of $1.03? We find it from the proportion:

$1 : $1.03 : : $15.373564 : the answer,

which is equal to 15.373564×$1.03=$15.8347709.

And this is the present value, at age forty-three, of a life series of annual payments of $1.03; and this $1.03 is the difference between the net annual premium at age forty-three and the net annual premium at age forty-two; and if the Company has the $15.83 on hand in deposit, which is the cash equivalent of this difference in the future net annual premiums; this amount of cash in hand, together with the smaller net annual premium due to the age forty-two, is just the same value as the net annual premium due to age forty-three. This $15.83 is the amount that must be held on deposit in trust for the policy of one thousand dollars taken out at age forty-two, at the end of the first year of the policy; and if the Company has it on hand, and keeps it securely invested at the net rate of interest, and regularly compounds the interest yearly, this "Trust Fund Deposit," together with the present value of the future net annual premiums, will always keep the policy that is paying the smaller net annual premiums due to the younger age at which the holder entered the Company, just on a par with those policies that come in later, or at a more advanced age of entry, and pay the larger annual premium due to this advanced age.

Now, let us take up again the symbols used in Life Insurance, and see how the formula is deduced by which the amount of the Trust Fund Deposit is, in practice, actually calculated.

To Determine a Formula for Computing the Deposit.

The present value of a future life series of annual payments of one dollar is represented by A; at any age x, it is A_x; and at the end of any number of years n, from the date of the policy, it is represented by A_{x+n}.

Now suppose a life policy for one dollar was taken out at the age x. What amount of trust fund must be in the hands of the Company to the credit of that policy at the end of n years from its date?

The net annual premium at the age x, is aP_x. The net annual premium at the age $x+n$, is aP_{x+n}. The difference between the two is $aP_{x+n}—aP_x$. The present value of this future life series of net annual premiums at age $x+n$, is found by the following proportion: A future life series of net annual premiums of one dollar, at the age $x+n$, is to its corresponding present value, which is A_{x+n}, as the future life series of net annual premiums ($aP_{x+n}—aP_x$), at the age $x+n$, is to its present value,

or $\$1 : A_{x+n} :: (a\,P_{x+n}—aP_x)$: the answer.

It is, therefore, $A_{x+n}(aP_{x+n}—aP_x)$. This expression gives the present value, at age $x+n$, of a future life series of annual premiums ($aP_{x+n}—aP_x$), and this is the amount that must be on hand, in trust, deposited to the credit of the policy at the end of n years from its date, in order to make, at that time, when added to the value of the future aP_x net annual premiums, an amount equivalent to the value, at that time, of the future aP_{x+n} net annual premiums due to the age $x+n$.

Another Formula for the Deposit.

Another expression may be obtained for the amount of Trust Funds on Deposit as follows: The net single premium, at any age $x+n$, minus the present value, at that age, of the future life series of aP_x annual premiums, is the amount that must be on hand in deposit to cause the future aP_x annual premiums and the deposit to be equivalent to the future aP_{x+n} annual premiums. From this we have the equation: $sP_{x+n}-(aP_x \times A_{x+n})=$ the *deposit* at the end of n years from the date of the policy. Substituting for sP_{x+n} its value, $1-(1-v)A_{x+n}$, and for aP_x its value, $\frac{1}{A_x}-(1-v)$, and we have $1-(1-v)A_{x+n}-\left(\frac{1}{A_x}-(1-v)\right)A_{x+n}=$the deposit at the end of n years. And as the two expressions $(1-v)A_{x+n}$, have different signs, they cancel each other, and we have, $1-\frac{A_{x+n}}{A_x}=$ "Deposit" at the end of n years. This is said to be, "perhaps, the easiest working formula for obtaining the amount of Trust Fund that should be on deposit for whole life policies."

To calculate by this formula, the "Deposit" at the end of the first year, for a whole life policy for $1,000 taken out at age 42, we have, in this case, $x=42$, $n=1$, and $x+n=43$; therefore, $1-\frac{A_{x+n}}{A_x}$, becomes $1-\frac{A_{43}}{A_{42}}$. But $A_{43}=\frac{N_{43}}{D_{43}}=\frac{216{,}841.2}{14{,}104.81}=15.373564$. And $A_{42}=\frac{N_{42}}{D_{42}}=\frac{231{,}671.7}{14{,}830.58}=15.621217$. Therefore, $1-\frac{A_{43}}{A_{42}}=1-\frac{15.373564}{15.621217}=1-0.984146=\$0.01585$. This is the "Deposit" for $1. Multiply it by 1,000, and we have $15.85 as the amount that must be on hand "deposited," at the end of the first year, to the credit of a whole life policy for $1,000, taken out at age 42.

It will be noticed that this value for the "Deposit" is two cents more than that previously obtained; which is due to the fact that the difference, $1.03, between the two net annual premiums, was not carried out to a further place of decimals.

Cost of Insurance.

The net Cost of Insurance, per year, properly chargeable to each policy, or, in other words, the proportion of "losses by death," that should be paid by each policy out of the net annual premium for the year, is obtained by multiplying the amount the Company has at risk upon the policy during the year, by the fraction that expresses the chance at the beginning of the year that the insured will die during the year. This fraction is obtained by dividing the number of deaths given by the table, for that year, by the whole number living at the beginning of the year. The result will give the amount that the policy should contribute out of the net annual premium for the year, to pay losses caused by death that year according to the Table of Mortality.

The "Amount at Risk" during any year, is equal to the amount of the policy minus the "Trust Fund Deposit" at the end of the year; because the net annual premium, having been paid at the beginning of the year, the Company has in its hands—of the policy-holders' money—the means for paying the "Cost of Insurance" during the year, properly chargeable to this policy; and also for providing the requisite Trust Fund Deposit at the end of the year. Therefore, the "Amount at Risk" during any year is the amount insured, minus the deposit at the end of the year.

Representing the "Trust Fund Deposit" by (T. F. D.), the "Deposit" at the end of *n* years from the date of the policy by $(\text{T. F. D.})_{x+n}$, we have the Deposit for the end of the next succeeding year represented by $(\text{T. F. D.})_{x+n+1}$. Subtracting this from one dollar, which is the amount insured, we have the "Amount at Risk" during the year expressed by $1-(\text{T. F. D.})_{x+n+1}$. Multiply this by the fraction which expresses the chance or probability of death during the year, and we obtain an expression for the amount chargeable to the policy for "Cost of Insurance" during the year. This fraction is obtained by taking from the table the number of deaths during the year, and dividing this number by the number living at the beginning of the year.

We are now considering the Mortality Table to be exact; the table or net interest always realized and compounded yearly; that the amount insured on each policy is one dollar; that there are enough policies at any given age to make the mortality amongst the insured conform to the general law of mortality called for by the table; and we are leaving out at present all consideration of either "expenses" or "profits." We have made calculations at net interest, to obtain a net annual premium that will exactly cover all the present chances, risks, and values, year by year, to the table limit of the duration of human life, ninety-nine years of age. Therefore, when the requisite "Deposit" for the end of the year has been set aside for the policy, and the net "Cost of Insurance" during the year properly chargeable to this policy has been paid, there should, under the assumptions made, be nothing left.

Let $\frac{d_{x+n}}{l_{x+n}}$= number of deaths during the year, divided by the number living at the beginning of the year. Then $\frac{d_{x+n}}{l_{x+n}}\left(1-(\text{T. F. D.})_{x+n+1}\right)$ will represent the cost of insurance paid by this policy during the year. $(\text{T. F. D.})_{x+n+1}$ represents the Deposit at the end of the year. The above are the two amounts to be paid or provided for.

At the beginning of each year there is on hand (or there ought to be on hand) the Deposit for the end of the preceding year; and the net annual premium for the coming year is paid. Both these amounts must be increased during the year by net or table interest; and out of these two sums thus increased, the net "Cost of Insurance" for the year must be paid, and the requisite Deposit for the end of the year must be provided.

Let $(\text{T. F. D.})_{x+n}(1+r)$ represent the Deposit on hand at the end of n years from the date of the policy, increased by net interest for one year.

Let $a\text{P}_x(1+r)$ represent the net annual premium, increased by net interest for one year; then, on the assumed data, we at once have the following equation:

$$(\text{T. F. D.})_{x+n}(1+r)+a\text{P}_x(1+r)-\frac{d_{x+n}}{l_{x+n}}\left(1-(\text{T. F. D.})_{x+n+1}\right)-(\text{T. F. D.})_{x+n+1}=0.$$

This is called "the equation of equitable balance."

Let us now Calculate the "Cost of Insurance," During the First Year, for a Whole Life Policy of $1,000, Taken out at Age Forty-two.

We have before found the "Deposit" at the end of the first year, for a whole life policy of one dollar, taken out at age forty-two, to be $0.01585. The "Amount at Risk" during the year will then be equal to $1—$0.01585=$0.984146.

The table shows the number of deaths during the year is 839, and the number living at the beginning of the year, 77,012; therefore, the fraction $\frac{839}{77,012}$ expresses the chance or probability that the insured will die during the year. This fraction, multiplied by the "Amount at Risk" during the year, gives us the "Cost of Insurance," during the year, properly chargeable to this policy.

Therefore, $\frac{839}{77,012} \times \$0.984146 = \$0.010721 =$ the "Cost of Insurance" on this policy of one dollar, during the year between age forty-two and age forty-three. Multiply this by 1,000, and we have the "Cost of Insurance" for the same year on a whole life policy for $1,000=$10.72. This, added to the amount requisite for the "Deposit" at the end of the year, which is $15.85, makes $26.57 that has to be paid or provided for. At the end of the preceding year there was no "Deposit," because the contract between the company and the insured had not been entered into. Therefore, for the year under consideration, the company has nothing but the net annual premium with which to pay the "Cost of Insurance" and provide the requisite "Deposit." We found the net annual premium to be $25.55. This is increased by net interest for one year. This interest has been assumed to be four per cent. per annum, and amounts to $1.02. This, added to the net annual premium, $25.55+$1.02=$26.57. The "Cost of Insurance" is paid and the "Deposit" provided for, and there is nothing left; $26.57 — $26.57 being equal to zero.

The "Reserve" Absolutely Necessary to Enable a Life Insurance Company to Pay its Policies at Maturity.

To illustrate the manner in which the "Deposit" must accumulate in the earlier years of a Life Insurance Company, in order to enable it to meet its obligations when the death claims exceed the premiums, let us suppose that a Company insures twenty thousand policy-holders, for five thousand dollars each, at age thirty. The net annual premium required from each person is $84.85. This, on 20,000 policies, would make the first payment of annual premiums amount to $1,697,000. The net interest is assumed to be four per cent., and, for the first year, it amounts to $67,880. The Company has, therefore, for the first year, $1,764,880. By the Table of Mortality 168 of the insured will die during the first year; to the heirs of each, the Company must pay five thousand dollars. The losses by death are, therefore, $840,000; leaving on hand with the Company, after all the death claims are paid, $924,880; which would be a handsome "*surplus*" at the end of the first year's business, but for the fact that every dollar of this sum belongs to the Trust Fund Deposit, and is an already ACCRUED LIABILITY—A DEBT.

At the end of the thirty-fourth year, the Deposit for each outstanding policy must be $2,464.25. The Table of Mortality shows that 11,297 of the policy-holders will be living at the end of the thirty-fourth year; the Company must, therefore, have on hand a Trust Fund Deposit amounting to $27,838,632.25. We find that 11,742 policy-holders were living at the beginning of the thirty-fourth year; and their net annual premiums amounted, in the aggregate, to $996,308.70. There were 445 deaths during the year, and the aggregate losses by death amounted to $2,225,000. Thus we see, in this year, the death claims exceed the annual premiums by more than one and one quarter millions of dollars. But the Company has on hand, in Deposit, at the end of the year $27,838,632.25, after having paid the death claims. The Company, however, is not rich, nor more than able to pay its liabilities, because it will surely take the last cent of this amount, and all the future net annual premiums, and compound interest regularly all the time, to enable it to meet and pay its now rapidly increasing death claims.

Let us look into the accounts of the Company at the end of the fiftieth year. The "Deposit" on account of each policy at the end of this year is $3,708.20; and there are living 3,080 policy-holders. The aggregate "Deposit" for the outstanding policies at this time, is $11,421,256.00. There were 461 deaths during the year, and the aggregate of policies that matured during the year amounted to $2,305,000. There were 3,541 policy-holders living at the beginning of the year, and the aggregate of the net annual premiums paid by them amounted to $300,453.85. We see from this that the losses by death during the year exceeded the net annual premiums by more than $2,000,000. The "Deposit" is reduced to $11,421,256.00, which is less than one half the amount in "Deposit" at the end of the thirty-fourth year. But the Company has not lost money, it has only been paying its debts. At the end of the thirty-fourth year it had more, but it owed more. It had enough then, and only enough, to pay what it owed; it is in the same condition now.

At the end of the sixty-fifth year, we find the "Deposit" that must be in the hands of the Company to the credit of each policy, is $4,560.87; and there are twenty of the original policy-holders living. The aggregate "Deposit" for these twenty outstanding policies is $91,217.40. The $27,838,632.25 that the Company had on hand at the end of the thirty-fourth year is now reduced to less than $100,000. But the Company has only been paying its debts to policy-holders—not losing money. In fact, it had none to lose *of its own.*

At the end of the sixty-ninth year, the "Deposit" amounts to $4,722.84; and there is *one* policy-holder living. He pays his regular net annual premium the day he is ninety-nine years old. The premium is $84.85. This, added to the "Deposit" on hand at the end of the preceding year, makes $4,807.69 of this policy-holder's money in the hands of the Company the day the policy-holder is ninety-nine years old. At net interest, which is four per cent., the interest for the year will amount to $192.31; and this, added to the amount $4,807.69, on hand at the beginning of the year, makes $5,000, with which to pay the policy of the last policy-holder in this Company.

We see that the $27,838,632.25, which the Company had in its possession at the end of the thirty-fourth year, belonging to the policy-holders, has been paid to them. The policies were all paid at maturity; the Company has nothing left. In fact, it never had a cent of its own during the whole time, although we have seen it the custodian, at one time, of nearly twenty-eight millions of dollars of other people's money. It owed every cent, and it paid every cent it owed.

It is a marked peculiarity of Life Insurance business, that the annual premiums exceed the death claims for the first thirty or forty years; after which time, the losses by death largely exceed the annual premiums. The Trust Fund Deposit is a fixed mathematical amount; it increases for each policy at the end of every succeeding year of the existence of the policy. And if the Life Insurance Company has, for each and every outstanding policy, the requisite "Deposit," it can pay its policies at maturity. This "Deposit," or "Great Sheet-Anchor of Life Insurance," is the "Sacred Fund" of the "Widows and Orphans," and ought to be guarded by wise and stringent laws, rigidly enforced by competent and honest officers of the State Government.

NOTE.—That the general reader may not suppose that an extreme in the amount of money involved has been assumed in the above hypothetical example, used merely for illustrating the necessity for "accumulation" during the earlier years of a Life Insurance Company, in order to meet its obligations when the death claims exceed the premiums, the fact is here mentioned that one Company in this country has now over 60,000 policies outstanding, insuring over $200,000,000; and its Trust Funds on Deposit amount to more than $30,000,000, although the Company is but twenty-seven years old.

COMMENTS ON AMOUNT IN DEPOSIT OR RESERVE.

Bear in mind that the "Deposit" must always be kept invested at the net or table rate of interest at least, and the interest must be regularly compounded every year, in order to enable the Company to pay its policies at maturity. It is, for this reason, not enough at the end of any year, that a Life Insurance Company should place to the credit of a policy the difference between the net annual premium due to the age at which the policy-holder entered the Company and that due to the age he has now attained; but the Company must place to the credit of the policy-holder an amount equal to the value in hand of a future life series of annual premiums, each of which is equal to the difference above referred to. It is thus seen that in Companies that have a large number of policies outstanding, and policies that have been in existence for a good many years, *the amount is large that must be, by the Company, placed to the credit of the policies, in addition to the annual premiums paid by policy-holders during the year in question*, IN ORDER TO MAKE THE POLICIES IN SUCH COMPANY SAFE.

The real practical meaning, therefore, of a "*Reserve*" amounting to $30,000,000 is this: The Company that has this amount in "Reserve" at the end of any year, has to pay thirty millions of dollars in addition to the annual premiums paid the following year by its policy-holders, before the amount is made up that will insure the payment of the policies at maturity.

It is absurd to speak of a large "Reserve" as evidence of riches or strength on the part of Life Insurance Companies. If the Company has the requisite "Reserve," or Trust Fund Deposit, it can meet its liabilities; if it has not the requisite *amount on* "*Deposit*," it is insolvent.

Although $30,000,000 in "*Reserve*" (being the amount in Trust on Deposit) does not indicate riches or excess of strength in a Life Insurance Company beyond the mere ability on the part of the Company to meet its accrued liabilities, IT MAKES THE COMPANY A GREAT FINANCIAL POWER, by the accumulation in the hands of its officers of this immense sum of money belonging to its policy-holders;

which amount may increase with an increase of its new business; may remain permanently at this sum if the new business in this respect counterbalances the excess of death claims over premiums upon its older policies; or, if the Company ceases to issue new policies, the Trust Funds on Deposit will in time be exhausted.

Registered Policies.

It is a matter of great importance to policy-holders to have the "Deposit" guarded against all chance of accident or loss. To effect this, some of the Companies issue "Registered Policies;" in which case, the Trust Fund for the policy, invested in safe interest-bearing stocks, or bonds and mortgages, is deposited with the State Treasurer; and the State becomes responsible to the policy-holder for the safe-keeping and proper application of the funds thus deposited. When the real nature and importance of this deposit or "Reserve," as it is generally called, and the vast aggregate amount that it must soon attain, are well understood by the general public, REGISTERED POLICIES will, no doubt, become more popular than they have been heretofore. THIS WOULD RELIEVE THE COMPANIES, IN GREAT DEGREE, FROM THE CARE AND RESPONSIBILITY ATTENDANT UPON THE HANDLING AND CONTROL OF THESE LARGE AMOUNTS OF OTHER PEOPLE'S MONEY.

In case a State is willing to become responsible for the safe-keeping, and to guarantee the proper application of this fund, it would seem that policy-holders should avail themselves of the security thus afforded. Not that the State guarantees the payment of the policy at maturity: this, I take it, a State will never do, unless it establishes a Government Life Insurance Company of its own. But some States have already agreed, and others possibly may hereafter agree, to become responsible for that portion of the fund of the Company generally known by the name "*Reserve;*" the same which in these Notes is called TRUST FUND DEPOSIT.

Whatever doubt there may be as to the proper course for a State to pursue in regard to this matter, there is hardly room for doubt that it is safer and better for the individual policy-holders to have their " TRUST FUND DEPOSIT," or " *Reserve*," guaranteed by the State, than to trust these vast sums solely to the officers of the Life Insurance Companies.

NO " DIVIDENDS " CAN BE MADE FROM THE NET ANNUAL PREMIUM AT NET INTEREST.

We have seen that the whole of the " net annual premium," at *net* interest, regularly compounded every year, is required to pay the " Cost of Insurance " during the year, and provide the "Deposit" at the end of the year, requisite to secure the payment of the policy at maturity. The " enormous dividends " made to policy-holders by Life Insurance Companies, and the large credits or loans so *generously* proffered, must, therefore, come from some source other than the "net annnal premium" increased by "net interest;" and it is equally clear that the so-called "Reserves," accumulated by Life Insurance Companies, cannot, with any propriety, be considered "cash capital." The foregoing remarks give an outline of the THEORY OF WHOLE LIFE INSURANCE.

ALGEBRAIC SUMMARY OF THE THEORY OF WHOLE LIFE INSURANCE.

Let v represent the amount of money that will, when increased by interest at the rate r, be equal to one dollar at the end of one year.

Then v, raised to a power, the exponent of which is n, will be the amount which will, at the same rate of interest r, be equal to one dollar at the end of n years—interest being compounded yearly.

Let l represent a number of persons living, and l_x represent the number living at the age x, and l_{x+1} the number of those that will be living at the age $x+1$, and so for other ages.

The amount v, at the rate of interest r, will be what it is now worth to insure one dollar, to be paid *certain* at the end of one year; and v^n is the amount that will insure one dollar, to be paid certain at the end of n years.

To insure one dollar, to be paid at the end of one year to the heirs of a person in case he dies during the year, it is necessary to multiply the amount that would, at a certain rate of interest, produce one dollar certain, at the end of one year, by the fraction which expresses the chance or probability that the insured will die during the year.

The amount that will produce one dollar certain at the end of one year is v. The fraction representing the chance of the insured dying during the year is expressed by the number of deaths during the year, divided by the number living at the beginning of the year. The number of deaths during the year is equal to the number living at the beginning of the year, minus the number living at the end of the year. The number of deaths, therefore, between the age x and the age $x+1$ is expressed by $l_x—l_{x+1}$; and the fraction expressing the chance or probability that the insured will die during the year, between the age x and the age $x+1$, is $\frac{l_x-l_{x+1}}{l_x}$. This, multiplied by v, gives $v\,\frac{l_x-l_{x+1}}{l_x}$ for the amount that will insure one dollar, to be paid to the heirs of a person aged x, in case he dies during the first year following the date of the transaction. In like manner we can determine what it is now worth to insure the person against dying during the second year, the third year, and any year—and every year, up to and including the table limit of the duration of human life; and by adding together these respective values for every year, we obtain what it is now worth to insure the one dollar for life. That is to say, we obtain the amount that will enable the Company to insure one dollar to be paid to the heirs of the insured at the end of any year in which he may die. Calling this amount or net single premium that will insure one dollar for life, at the age x, sP_x, we have—

$$sP_x = v\frac{l_x - l_{x+1}}{l_x} + v^2\frac{l_{x+1} - l_{x+2}}{l_x} + v^3\frac{l_{x+2} - l_{x+3}}{l_x} +, \text{\&c., to age 99};$$

or (1), $sP_x = v\dfrac{l_x + vl_{x+1} + v^2 l_{x+2} +, \text{\&c.}}{l_x} - \dfrac{vl_{x+1} + v^2 l_{x+2} + v^3 l_{x+3} +,}{l_x}$ &c.

Now let us obtain an expression for the present value, or sum in hand, that will, at any age x, be the precise money equivalent of a future life series of annual premiums or payments of one dollar; the first payment to be made in advance, and one at the beginning of each year following, provided the insured is alive to make the payment. The first payment, and therefore the first term of the series, is one dollar. This is paid in hand; is certain; is equal to unity or one dollar, and may be represented by $\frac{l_x}{l_x}$. The present value of one dollar, to be paid certain at the end of one year, is v; but the second payment, at the end of the first or beginning of the second year, is only to be made in case the insured is alive at the time the second payment is due. The chance or probability that the insured will be alive is obtained by dividing the number living at the age $x+1$ by the number living at the age x. Therefore, the present value of the second payment is expressed by $v\frac{l_{x+1}}{l_x}$. By similar reasoning, the present value of the third payment is $v^2\frac{l_{x+2}}{l_x}$; and so for the other payments.

Calling the value of all the payments A_x, we are at once enabled to write the equation:

$$A_x = \frac{l_x}{l_x} + v\frac{l_{x+1}}{l_x} + v^2\frac{l_{x+2}}{l_x} +, \text{\&c., to ninety-nine years of age,}$$

$$(2) \text{ or } A_x = \frac{l_x + vl_{x+1} + v^2 l_{x+2} +, \text{\&c.,}}{l_x} \text{ to ninety-nine.}$$

By combining equation (2) with equation (1) we have:

$$sP_x = vA_x - (A_x - 1);$$

$$(3) \text{ or, } sP_x = 1 - (1 - v)A_x.$$

The expression A_x, gives us the amount in hand, which is the precise money equivalent of a future life series of annual premiums of one dollar at the age x. Therefore A_x is to sP_x, as a future life series of annual premiums of one dollar is to a future life series of annual premiums that will be the precise money equivalent of the amount sP_x in hand; from which we deduce the annual premium which is the equivalent of sP_x in hand: it is $\frac{sP_x}{A_x}$; and as sP_x in hand will insure one dollar for life at the age x, therefore its equivalent, in money value—an annual premium amounting to $\frac{sP_x}{A_x}$: will insure one dollar for life at the age x. Calling this net annual premium aP_x, we have:

$$(4)\ aP_x = \frac{sP_x}{A_x} = \frac{1}{A_x} - (1 - v).$$

The net annual premium at any age x being aP_x; that at age $x+n$ will be aP_{x+n}, and the net single premium, at age $x+n$, will be sP_{x+n}. The value of a policy at the age $x+n$, or the net single premium that will purchase a whole life policy of one dollar at the age $x+n$, is sP_{x+n}. The value at age $x+n$ of a future life series of annual premiums, each equal aP_x, is represented by $aP_x \times A_{x+n}$.

The "Deposit" at the end of the n^{th} year of a whole life policy, taken out at the age x, represented by $(T.F.D.)x+n$, must be sufficient to make, at the age $x+n$, when added to the value, at that age, of the future life series of aP_x net annual premiums, an amount equal to the net single premium at that age. We therefore have:

$$sP_{x+n} - aP_x \times A_{x+n} = (T.\ F.\ D.)x+n;$$

From equation (3) we deduce $sP_{x+n}=1-(1-v)A_{x+n}$. Therefore $(T.\ F.\ D.)_{x+n}=1-(1-v)A_{x+n}-aP_x\times A_{x+n}$.

But $aP_x=\frac{sP_x}{A_x}=\frac{1}{A_x}-(1-v.)$ See equation (4.) Therefore, $(T.\ F.\ D.)_{x+n}=1-(1-v)A_{x+n}-\left(\frac{1}{A_x}-(1-v)\right)A_{x+n}$;

$$(5) \text{ or } (T.\ F.\ D.)_{x+n}=1-\frac{A_{x+n}}{A_x}.$$

We have previously shown how to obtain the value of A at any age. The fraction expressing the value is found by taking the numerator and denominator in the N_x and D_x columns of the table, opposite the designated age. Having obtained by this means the value of A, the net single premium (see equation 3) is expressed by:

$$sP_x=1-(1-v)A_x.$$

The net annual premium $aP_x=\frac{1}{A_x}-(1-v.)$ (See eq. 4.)

And the "Deposit" $(T.\ F.\ D)_{x+n}=1-\frac{A_{x+n}}{A_x}$. (See eq. 5.)

Having obtained the requisite "Deposit" that must be on hand at the end of any year; by subtracting this from the amount of the policy, we have the amount at risk during that year. Multiply the amount at risk during any year by the fraction which represents the chance or probability that the insured will die during the year, and the result will give the "Cost of Insurance" during that year.

The net annual premium at net interest for the first year will be exactly sufficient to pay the net Cost of Insurance during the year, and provide the "Deposit" for the end of the year.

The net annual premium paid at the beginning of the second year, added to the "Deposit" at the end of the first year, will produce a sum which, increased by net interest, will exactly pay the "Cost of Insurance" during the second year, and provide for the "Deposit" that must be on hand at the end of the second year; and in like manner for the third year, fourth, and every year up to, and including, the limit of the duration of human life, according to the law of mortality expressed by the table used.

Thus the net annual premium paid the last year of the table will, when added to the "Deposit" for the end of the preceding year, make a sum which, at net interest for one year, will exactly amount to the face of the policy, and this, too, after paying "Cost of Insurance," properly chargeable to this policy in every previous year.

From what precedes, it is seen that at any age x, the amount is easily calculated that will, if paid in hand, insure one dollar to be paid to the heirs of a person at the end of any number of years indicated by n, in case the insured dies during the n^{th} year. By making this calculation for every year, to the limit of the Table of Mortality, and adding together all these respective yearly results, we obtain the amount that will, if paid in hand, insure the one dollar to be paid to the heirs at the end of any year in which the insured may die.

The amount that will insure one dollar, in case the insured dies during the first year, is very small when x is small. For instance, at age twenty, it requires, to insure one dollar, between the age twenty and the age twenty-one, an amount equal to v, multiplied by the fraction which expresses the chance or probability that the insured may die during the year; or, $\$0.961538 \times \frac{680}{93,268} = \$0.0070104$. For a policy of $1,000 the amount required is $7.01. This is cheap for the first year; but must be paid in cash in advance. Bear in mind that this method provides no "Deposit" for the payment of policies that mature in the future.

The net annual premium at the same age, to insure $1,000 for whole life, is $12.95; but this amount will, in addition to paying the "Cost of Insurance" year by year, furnish the means for providing the requisite "Deposit," and make the ultimate payment of the policy at maturity CERTAIN So that at age ninety-nine, in case the insured is still living, he will, at that age, only have to pay $12.95 to effect his insurance during the one hundredth year of his age. Let us see how the matter will stand at age ninety-nine with a person who has no "Deposit" accumulated from year to year. At age ninety-nine the amount requisite to insure one dollar for one year, is v multiplied by the fraction which expresses the chance or probability that the insured will die during the one hundredth year of his age; or, $v \times \frac{1}{1} = v =$ \$0.961538. To insure \$1,000 for one year at the same age will require \$961.54.

4

INSURANCE OTHER THAN WHOLE LIFE.

VALUE OF n ANNUAL PAYMENTS OF ONE DOLLAR EACH.

We have previously seen that:

$$A_x = \frac{v^x l_x + v^{x+1} l_{x+1} + v^{x+2} l_{x+2} +, \text{\&c., to age ninety-nine;}}{v^x l_x}$$

and that this expression assumed the form:

$$A_x = \frac{D_x + D_{x+1} + D_{x+2} + \ldots \text{ to } D_{99}}{D_x} = \frac{N_x}{D_x}.$$

Suppose that it is desired to find an expression for the present value of a series of n annual payments of one dollar each, provided the person is alive to make the payments, the first being made in advance at the age x.

It is clear that if we take the first n terms of the second member in either of the two equations above, and make the calculations indicated, we will obtain the desired result. But by using the N_x and D_x columns of the table, the arithmetical operation can be very much shortened. We find the value of N_x from the table by taking the number in that column opposite the age x. In doing this, we have taken the whole series of annual payments of one dollar to the limit, ninety-nine years; but this is too much, because we want only the first n terms. We have, therefore, to find the value of the terms not included in the first n terms, and subtract their sum from the sum of all the terms in N_x, which will give us the present value of the first n terms. The first payment is made in hand at the age x; the second is made at the age $x+1$, and n payments will have been made at the age $x+n-1$. All the terms following this last are to be subtracted from the N_x series; that is to say, we commence at the age $x+n$ and take the series from that term to the limit, ninety-nine years, and subtract the sum of this series of terms from that obtained by starting at the age x, and taking the series through to age ninety-nine. The latter is expressed by N_x; the former by N_{x+n}. Therefore, the present value of one dollar, payable annually for n years, provided the person is alive to pay it—the first payment being made in advance; the age of the person being x years—is expressed by $\frac{N_x - N_{x+n}}{D_x}$. Calling this $A_{x|n}$, we have $A_{x|n} = \frac{N_x - N_{x+n}}{D_x}$. The symbol $A_{x|n}$ indicates that the first n payments only of one dollar are taken:

EXAMPLE.

At age thirty, what is the value of a series of twenty annual payments of one dollar each, the first payment to be made in advance, and one at the beginning of each succeeding year, provided the person is alive to make the payment?

$$A_{x|n} = \frac{N_x - N_{x+n}}{D_x}. \quad x=30. \quad n=20.$$

$$\text{Then } \frac{N_x - N_{x+n}}{D_x} = \frac{N_{30} - N_{50}}{D_{30}}.$$

$N_{30}=479951.6.$ $N_{50}=131765.6.$

$N_{30}-N_{50}=348186.0.$ $D_{30}=26{,}605.37.$

$$\frac{N_{30}-N_{50}}{D_{30}} = \frac{348186.0}{26605.37} = \$13.087057.$$

VALUE OF A LIFE SERIES OF ANNUAL PAYMENTS OF ONE DOLLAR EACH, THE FIRST PAYMENT TO BE MADE AT THE END OF n YEARS.

The symbol nA_x is used to represent that the payments of the one dollar are postponed n years, and then commence and continue for life. In this case we have only to omit the first n terms of the numerators in the expression for A_x, and we have: $nA_x = \frac{N_{x+n}}{D_x}$.

EXAMPLE.

What is the value, at age forty, of a life series of annual payments of one dollar each, the first payment to be made at the end of thirty years, and continue for life?

$$nA_x = \frac{N_{x+n}}{D_x}. \quad x=40. \quad n=30.$$

$$\text{Then } nA_x = \frac{N_{70}}{D_{40}} = \frac{16840.03}{2301.43} = \$7.317202.$$

Term Insurance.

Let us suppose it is desired to find an expression for the net single premium that will insure one dollar for a term of years only. Let the number of years, as before, be represented by n; we have before found

$$sP_x = v\frac{l_x + vl_{x+1} + v^2 l_{x+2} + \ldots}{l_x} - \frac{vl_{x+1} + v^2 l_{x+2} + v^3 l_{x+3} +, \&c.}{l_x}$$

Multiplying both the numerator and denominator of each of the above fractions by v^x, we have—

$$sP_x = v\frac{v^x l_x + v^{x+1} l_{x+1} + v^{x+2} l_{x+2} + \ldots}{v^x l_x} \ldots$$

$$-\frac{v^{x+1} l_{x+1} + v^{x+2} l_{x+2} + v^{x+3} l_{+3x} +, \&c.}{v^x l_x},$$ which becomes,

as the series in this case extends to ninety-nine years:

$$sP_x = \frac{vN_x}{D_x} - \frac{N_{x+1}}{D_x}.$$

Observe that the first term in the series, contained in the numerator of the negative term of the second member of the above equation, begins with the year $x+1$.

The above expression for sP_x is the net single premium that will insure one dollar for life; but we want to find an expression for the first n years only. We must, therefore, subtract from the above expression the value of all the terms of this series after passing the first n terms. Leaving out these first n terms, the sum of the series, from age $x+n$ to age ninety-nine, will be expressed by $v\frac{N_{x+n}}{D_x} - \frac{N_{x+n+1}}{D_x}$, and subtracting this from the whole series for life, as given above, and calling the net single premium that will insure one dollar for n years at age x, $sP_{x|n}$ we have:

$$sP_{x|n} = v\frac{N_x}{D_x} - \frac{N_{x+1}}{D_x} - \left(v\frac{N_{x+n}}{D_x} - \frac{N_{x+n+1}}{D_x}\right)$$

$$\text{Or, } sP_{x|n} = \frac{v(N_x - N_{x+n}) - (N_{x+1} - N_{x+n+1})}{D_x}.$$

This gives the value of the net single premium that will insure one dollar for a term of n years.

EXAMPLE.

What net single premium is required at age thirty to insure $1,000 *for twenty years?*

$$sP_{x|n} = \frac{v(N_x - N_{x+n}) - (N_{x+1} - N_{x+n+1}.)}{D_x}$$

$N_x = N_{30} = 479951.6.$ $N_{x+n} = N_{50} = 131765.6.$

$N_{30} - N_{50} = 348186.0.$ $v(N_{30} - N_{50}) = 334794.07.$

$N_{x+1} = N_{31} = 453346.2.$ $N_{x+n+1} = N_{51} = 121983.7.$

$N_{31} - N_{51} = 331362.5.$

$v(N_{30} - N_{50}) - (N_{31} - N_{51}) = 3431.57.$ $D_x = D_{30} = 26605.37.$

Therefore $sP_{x|n} = sP_{30|20} = \frac{3431.57}{26605.37} = \$0.1289803.$

This will insure one dollar as above. Multiply by 1,000, and we have the result required—$128.98.

NOTE.—*In case the net single premium has been calculated as above, and the numerical value of* $A_{x|n}$ *corresponding thereto, has been determined; the net annual premium is obtained by dividing* $sP_{x|n}$ *by* $A_{x|n}$.

NET ANNUAL PREMIUM FOR THE ABOVE POLICY.

The amount $A_{x|n}$ in hand is the equivalent of an annual premium of one dollar for n years, therefore $sP_{x|n}$ in hand is the equivalent of a proportional annual premium for n years. Hence—

$$A_{x|n} : sP_{x|n} :: \$1 : \frac{sP_{x|n}}{A_{x|n}}.$$

This fourth term is an expression for the net annual premium that will insure one dollar for n years. By substituting for $sP_{x|n}$ and $A_{x|n}$ in this expression, their values, as obtained above, we have—

$$\frac{sP_{x|n}}{A_{x|n}} = \frac{v(N_x - N_{x+n}) - (N_{x+1} - N_{x+n+1})}{N_x - N_{x+n}} = v - \frac{N_{x+1} - N_{x+n+1}}{N_x - N_{x+n}}.$$

This is the formula that is used in calculating the net annual premium that will insure one dollar for a term of n years.

Example.

What is the net annual premium that will, at age thirty, insure $1,000 *for twenty years?*

$$\frac{sP_{x|n}}{A_{x|n}}=v-\frac{N_{x+1}-N_{x+n+1}}{N_x-N_{x+n}}=v-\frac{N_{31}-N_{51}}{N_{30}-N_{50}}.$$

$$N_{31}-N_{51}=331362.5. \quad N_{30}-N_{50}=348186.0.$$

$$\frac{N_{31}-N_{51}}{N_{30}-N_{50}}=\frac{331362.5}{348186.0}=\$0.951682.$$

$$v-\frac{N_{31}-N_{51}}{N_{30}-N_{50}}=\$0.961538-\$0.951682=\$0.009856.$$

This is the net annual premium that will insure one dollar as above. Multiply by 1,000, and we have $9.856, which is the net annual premium that will insure $1,000 as above.

Endowment.

By an "Endowment," money is assured to be paid in case the person insured is living at the time named in the endowment policy, say n years from its date. The present probability of his being alive at the age $x+n$ is equal to the fraction obtained by dividing the number of those living at the age $x+n$ by the number living at the age x. This fraction multiplied by the present value of one dollar, to be paid certain at the end of n years, will give the present value of an endowment of one dollar, that is: $\frac{v^n l_{x+n}}{l_x}$. But this is equal to $\frac{v^{x+n} l_{x+n}}{v^x l_x}=\frac{D_{x+n}}{D_x}$.

Representing the net single premium for an endowment of one dollar at age x, payable in n years, by $E_{x|n}$, we have the equation $E_{x|n}=\frac{D_{x+n}}{D_x}$.

Example.

What is the net single premium that will, at age thirty, insure an endowment of $1,000, *to be paid at age fifty?*

$$E_{x|n}=\frac{D_{x+n}}{D_x}=\frac{D_{50}}{D_{30}}=\frac{9781.92}{26605.37}=\$0.367667.$$

This is the net single premium that will insure one dollar. Multiply by 1,000, and we have \$367.66, which is the net single premium that will, at age thirty, insure an endowment of \$1,000, to be paid at age fifty, in case the insured is alive at that time.

Net Annual Premium for this Endowment.

$A_{x|n}$ in hand is the equivalent of an annual premium of one dollar for n years, and the net annual premium in this case is expressed by $\frac{E_{x|n}}{A_{x|n}}$. Substituting in this expression for $E_{x|n}$ its value, $\frac{D_{x+n}}{D_x}$, and for $A_{x|n}$ its value, $\frac{N_x - N_{x+n}}{D_x}$, the expression becomes $\frac{E_{x|n}}{A_{x|n}} = \frac{D_{x+n}}{N_x - N_{x+n}}$.

This is the formula used for calculating the net annual premium for an endowment to be paid at the end of n years.

Example.

What is the net annual premium that will insure an Endowment of \$1,000 as above?

$$\frac{E_{x|n}}{A_{x|n}} = \frac{D_{x+n}}{N_x - N_{x+n}} = \frac{D_{50}}{N_{30} - N_{50}} = \frac{9781.92}{348186.0} = \$0.028094.$$

This will insure one dollar as above. Multiply by 1,000, and we have \$28.09, which is the net annual premium at age thirty for an endowment of \$1,000, to be paid in twenty years, or at age fifty—in case the insured is alive at that time to receive the endowment.

Endowment and Term Insurance Combined.

In case of endowment, payable at age $x+n$, and insurance payable at death if previous, the net single premium for the endowment is added to the net single premium for the term insurance, and we have the expression: $sP_{x|n} + E_{x|n}$. Substituting for each of these expressions their respective values as given above, and observing that $D_{x+n}+N_{x+n+1}$ is equivalent to N_{x+n}, we have the equation:

$$(sP+E)_{x}|_{n}=\frac{N_{x+n}+v(N_x-N_{x+n})-N_{x+1}}{D_x}.$$

EXAMPLE.

What is the net single premium that will, at age thirty, insure \$1,000 *to be paid in case the insured dies within twenty years, and at the same time insure an endowment of* \$1,000, *to be paid at the end of twenty years, in case the insured is alive at that time?*

$$(sP_{+}E)_{x}|_{n}=\frac{N_{x+n}+v(N_x-N_{x+n})-N_{x+1}}{D_x}=\frac{N_{50}+v(N_{30}-N_{50})-N_{31}}{D_{30}}.$$

$v(N_{30}-N_{50})=334794.07$; add N_{50} and we have:

$334794.07+131765.6=466559.67$; subtract

N_{31} from this and we have: 13213.47; and

$$\frac{N_{50}+v(N_{30}-N_{50})-N_{31}}{D_{30}}=\frac{13213.47}{26605.37}=\$0.496646.$$

Multiply this result by \$1,000, and it gives \$496.65, which is the required net single premium.

NET ANNUAL PREMIUM FOR THE ABOVE.

The net annual premium is equal to $\frac{(sP+E)_{x}|_{n}}{A_{x}|_{n}}$; and by substituting for the numerator and denominator their respective equivalent expressions, as given above, we have $v-\frac{N_{x+1}-N_{x+n}}{N_x-N_{x+n}}$. This is the formula used for calculating the net annual premium for a policy combining Endowment and Insurance of one dollar, payable in n years, or at death, if prior. This is usually called "Endowment," simply.

EXAMPLE.

To find the corresponding net annual premium.

$$\frac{(sP+E)_{x}|_{n}}{A_{x}|_{n}}=v-\frac{N_{x+1}-N_{x+n}}{N_x-N_{x+n}}=v-\frac{N_{31}-N_{50}}{N_{30}-N_{50}}=v-\frac{321580.6}{348186.0}=$$

$v-\$0.923588=\$0.961538-\$0.923588=\$0.037950$. Multiply this by 1,000, and we have \$37.95, the required net annual premium.

Whole Life Insurance Paid for in n Years.

To find the formula used in calculating the net annual premium for whole life insurance, payable by n annual premiums, we use again the proportion: as $A_{x|n}$, in hand, is the equivalent of n annual payments of one dollar, any other amount, in hand, would be the equivalent of a proportional annual payment for n years. We have already seen that sP_x, in hand, is the net single premium that will insure one dollar for life—hence, the proportion:

$$A_{x|n} : sP_x : : \$1 : \frac{sP_x}{A_{x|n}}.$$

The fourth term of this proportion is the net annual premium for n years that will be equivalent to sP_x in hand; but sP_x in hand is the net single premium that will insure one dollar for life; therefore, $\frac{sP_x}{A_{x|n}}$ is the net annnal premium for n years that will insure one dollar for life. It has before been seen that $sP_x = 1-(1-v)A_x = 1-(1-v)\frac{N_x}{D_x} = \frac{D_x-(1-v)N_x}{D_x}$; and $A_{x|n}$ is equal to $\frac{N_x - N_{x+n}}{D_x}$; therefore, $\frac{sP_x}{A_{x|n}} = \frac{D_x-(1-v)N_x}{N_x - N_{x+n}}$. This is the formula used for calculating the net annual premium for n years that will insure one dollar to be paid to the heirs of the policy-holder at the end of the year in which he may die.

Example.

What is the net annual premium for ten years, at age forty, that will insure \$1,000 for life?

$$\frac{sP_x}{A_{x|n}} = \frac{D_x-(1-v)N_x}{N_x-N_{x+n}} = \frac{D_{40}-(1-v)N_{40}}{N_{40}-N_{50}}.$$

$D_{40}=16382.56$; $(1-v)=\$0.038462$; $N_{40}=263643.5$.

$N_{50}=131765.6$. From which we obtain—

$$\frac{D_{40}-(1-v)N_{40}}{N_{40}-N_{50}} = \frac{1638256-1014025629}{263643.5-131765.6} = \frac{6242.30371}{131877.9} =$$

0.04733. Multiply by 1,000, and we have \$47.33, the net annual premium required to insure \$1,000 for life, at age forty, in ten annual premiums.

GENERAL FORMULA FOR THE TRUST FUND DEPOSIT.

The net annual premium may be considered as composed of two separate and distinct parts, each part accurately adjusted to accomplish a specific purpose. One portion of the net annual premium is intended to pay losses that will occur each year by the death of a certain number of policy-holders, as indicated by the Table of Mortality; the other portion of the net annual premium is intended to provide the amount that the Company must hold, at the end of each year, on deposit, in trust for the policy-holder. Of course, in order to accomplish these purposes, each of the respective parts or portions into which we assume that the net annual premium may be divided, must be increased by net interest.

The value of the risk for one year, at age x, on one dollar, has been shown to be $v\frac{d_x}{l_x}$. That is to say, $v\frac{d_x}{l_x}$ will, when increased by net interest for one year, be the amount that will pay the risk on one dollar. The net annual premium is more than the actual value of the risk on one dollar for the year. This excess in the payment makes the actual "Amount at Risk" always less than the amount of the policy. The "Amount at Risk" during the first year, on a policy of one dollar, has been shown to be equal to $\$1—(\text{T. F. D.})_{x+1}$.

The value of the risk on one dollar being $v\frac{d_x}{l_x}$, the actual risk on the amount $\$1—(\text{T. F. D.})_{x+1}$, is expressed by $v\frac{d_x}{l_x}\left(1—(\text{T. F. D.})_{x+1}\right)$. Subtract this from the net annual premium, and we have that portion of the net annual premium that is intended to provide the requisite amount to be held on deposit in trust, viz: $aP_x—v\frac{d_x}{l_x}\left(1—(\text{T. F. D.})_{x+1}\right)$ Add to this net interest for one year, and we have the amount that must be held on deposit in trust for the policy-holder at the end of the first year of the policy.

Let the "*ratio*" of interest be represented by r. Observe that this r is *not the rate of interest;* it is a quantity which will, when the principal is multiplied by this quantity, produce what the principal will amount to when increased by net interest for one year. For instance, v being the principal and r the ratio of interest, rv is equal to one dollar; *and of course r is equal to one divided by v.*

All that is now to be done in order to form an equation that will enable us to determine the amount that must be held by the Company at the end of the first year on deposit in trust for the policy-holder, is to write the expression we use to represent this amount, viz: $(\text{T. F. D.})_{x+1}$, and place it equal to what that portion of the net annual premium not required to pay losses by death, viz:

$$aP_x - v\frac{d_x}{l_x}\left(1 - (\text{T. F. D.})_{x+1}\right)$$

will become, when net interest for one year is added to it.

This amount, resulting from adding the interest to the principal, is obtained by multiplying the principal by the "*ratio*" of interest, which is r; therefore—

$$(\text{T. F. D.})_{x+1} = r\left(aP_x - v\frac{d_x}{l_x}(1 - (\text{T. F. D.})_{x+1.})\right)$$

This is a simple equation of the first degree, containing only one unknown quantity, and that is the quantity sought for, viz: the amount on deposit in trust at the end of the first year.

It is true the unknown quantity is found in both members of the equation; but that is usual in the simplest elementary problems in algebra. We have used the above notation because it was supposed to be easy for the general reader to retain the idea that $(\text{T. F. D.})_{x+1}$ means "*the amount the Company must hold, on deposit, in trust for the policy-holder, at the end of the first year of a policy taken out at the age x.*"

Suppose we had called this amount y, and had represented the net annual premium that will insure one dollar for life at the age x, by p; and that the present value of one dollar, to be paid certain at the end of one year, multiplied by the fraction which represents the chance that the policy-holder will die during the first year, is represented by b. Then r, p, and b are known quantities, and the numerical value of each is easily determined.

The above equation, when written with the symbols just assumed, will become, $y=r(p-b(1-y))$; or $y=rp-rb+rby$; or $y-rby=r(p-b)$; or $y(1-rb)=r(p-b)$; hence $y=\frac{r}{1-rb}(p-b)$. As these quantities r, p, and b, are easily calculated, and their arithmetical value readily determined, it is clear that there can be no very abstruse mathematics required in the solution of this question.

This digression was entered upon for the purpose of illustrating the fact, that no very "*high order of mathematical attainment*" is essential to a clear comprehension of the principles and formula used in Life Insurance calculations.

We will resume the subject of the "amount that must be on deposit, in trust, at the end of the first year," by again writing the general equation:

$$(\text{T. F. D.})_{x+1}=r\left(a\text{P}_x-v\frac{d_x}{l_x}(1-(\text{T. F. D.})_{x+1})\right)$$

Call $v\frac{d_x}{l_x}$, c_x, and the equation becomes,

$$(\text{T. F. D.})_{x+1}=r\left(a\text{P}_x-c_x(1-(\text{T. F. D.})_{x+1})\right);$$

$$\text{or } (\text{T. F. D.})_{x+1}=ra\text{P}_x-rc_x+rc_x(\text{T. F. D.})_{x+1};$$

$$\text{or } (\text{T. F. D.})_{x+1}-rc_x(\text{T. F. D.})_{x+1}=ra\text{P}_x-rc_x;$$

$$\text{or } (1-rc_x)(\text{T. F. D.})_{x+1}=r(a\text{P}_x-c_x);$$

$$\text{hence } (\text{T. F. D.})_{x+1}=\frac{r}{1-rc_x}(a\text{P}_x-c_x).$$

Call $\frac{r}{1-rc_x}$ u_x, and we have,

$$(\text{T. F. D.})_{x+1}=u_x(a\text{P}_x-c_x.)$$

This is the formula used in calculating the amount that should be held by a Life Insurance Company on deposit in trust at the end of the first year of a policy taken out at the age x.

To obtain the formula used in calculating the amount on deposit at the end of two years, we first find the value of the risk on one dollar during the second year. This is expressed by $v\frac{d_{x+1}}{l_{x+1}}$, or c_{x+1}. The actual "Amount at Risk" during the second year is equal to $1-(\text{T. F. D.})_{x+2}$. Therefore, the actual value of the real risk during the second year, is $c_{x+1}\left(1-(\text{T. F. D.})_{x+2}.\right)$ Taking this from aP_x, we obtain that portion of the net annual premium paid the second year, that is applied towards forming the amount that must be held in deposit at the end of that year. Bear in mind that the amount in deposit at the end of the first year is exclusively used in making up the amount that must be held in deposit at the end of the second year. We are, therefore, enabled to write the equation $(\text{T. F. D.})_{x+2}=r\left((\text{T. F. D.})_{x+1}+aP_x-c_{x+1}(1-(\text{T. F. D.})_{x+2}.)\right)$ By transposing as before, and finding the value of the unknown quantity $(\text{T. F. D.})_{x+2}$, and calling $\frac{r}{1-rc_{x+1}}$ u_{x+1}, we obtain the following equation:

$$(\text{T. F. D.})_{x+2}=u_{x+1}\left((\text{T. F. D.})_{x+1}+aP_x-c_{x+1}.\right)$$

By similar reasoning we are enabled to write out the general expression or formula for obtaining the amount that must be, by the Life Insurance Company, held on deposit, in trust for the policy-holder, at the end of n years from the date of his policy, taken out at the age x. It is as follows:

$$(\text{T. F. D.})_{x+n}=u_{x+n-1}\left((\text{T. F. D.})_{x+n-1}+aP_x-c_{x+n-1}.\right)$$

This formula applies in calculating the amount that must be held in deposit at the end of n years from the date of the policy; and is general, being applicable to all kinds of policies. The formula deduced previously is used only in determining the amount on deposit for whole life policies. It will be noticed that, by the latter, the calculations can be made for any named year, without reference to the amount on deposit any previous year. The general formula deduced above requires that the amount on deposit at the end of the year just preceding should be known, before the amount for the end of the year in question can be calculated.

Tables have been constructed containing the values of ux and cx at the different ages.

EXAMPLE.

WHAT IS THE AMOUNT THAT MUST BE HELD BY THE COMPANY ON DEPOSIT, IN TRUST FOR THE POLICY-HOLDER, AT THE END OF THE FIRST YEAR, ON A TWENTY YEARS' "ENDOWMENT AND INSURANCE" POLICY FOR $1,000, TAKEN OUT AT AGE THIRTY?

$(\text{T. F. D.})_{x+1} = u_x(a\text{P}_x - c_x.)$ As before calculated, $a\text{P}_x =$ \$0.03795. From the table $u_x = 1.04884$, and $c_x = 0.008101$. Therefore $(\text{T. F. D.})_{x+1} =$ \$0.03131. Multiply by 1,000, we have \$31.31, which is the amount that must be on deposit at the end of the first year on the above policy.

TO DETERMINE THE AMOUNT THAT MUST BE HELD BY THE COMPANY AT THE END OF THE SECOND YEAR ON DEPOSIT, IN TRUST FOR THE POLICY-HOLDER, ON THE ABOVE ENDOWMENT AND INSURANCE POLICY.

We have, as before, $a\text{P}_x =$ \$0.03795: $(\text{T. F. D.})_{x+1}$, as above calculated, is \$0.03131; $u_{x+1} = 1.04900$ from the table; and $c_{x+1} = 0.008248$; this is also taken from the table.

We substitute these numerical values in the formula, and it becomes—

$(\text{T. F. D.})_{x+2} = 1.04900\ (\$0.03131 + \$0.03795 - \$0.008248)$.

Performing the arithmetical operations indicated, we have $(\text{T. F. D.})_{x+2} =$ \$0.64001588; multiply by 1,000, and we have \$64.00. This is the amount that must be in deposit for the above policy at the end of the second policy year.

AMOUNT OF THE DEPOSIT IN CASE THE POLICY IS PAID FOR BY A NET SINGLE PREMIUM.

In case a policy is paid for by a net single premium, at any age x, at the end of the first policy year, the Company must have on hand, deposited to the credit of the policy, after paying the net cost of insurance during the year, an amount that will be sufficient to make the payment of the policy at maturity safe.. In other words, the amount on hand in deposit, at the end of the first policy year, must be equal to the net single premium at the age $x+1$; and, in like manner, the deposit at the end of n years must be equal to the *net* single premium for the year $x+n$.

ANOTHER FORMULA FOR THE AMOUNT ON DEPOSIT

May be obtained as follows: The number of persons living, as given in the Mortality Table, at the age $x+n$, is expressed by l_{x+n}. Suppose all these persons are insured for one dollar each. What is the amount on deposit for each policy-holder living at the end of the year $x+n+1$?

The amount on deposit for each policy-holder, at the end of n years, is expressed by $(\text{T. F. D.})_{x+n}$. At the beginning of the $n^{th}+1$ year, each policy-holder pays his net annual premium, represented by aP_x. To obtain the net funds of this Company for the year, we add the net annual premium to the amount on deposit; multiply this sum by the number of policy-holders living, and add net interest for one year.

The "*ratio*" of interest is r; therefore, $\left((\text{T. F. D.})_{x+n}+aP_x\right)\times l_{x+n}\times r$ is an expression for the whole net funds, with interest for the year added. Out of this the cost of insurance for the year must be paid. The number of deaths during the year is d_{x+n}. Each policy is one dollar. Therefore d_{x+n} represents the cost of insurance for the year in question. Subtract this from the net funds on hand, and we have the whole amount that the Company must hold on deposit in trust for its policy-holders at the end of $n+1$ years. This amount on deposit for each policy-holder is expressed by $(\text{T. F. D.})_{x+n+1}$. The number of policy-holders living at the age $x+n+1$, is expressed by l_{x+n+1}. Therefore,

$$l_{x+n+1}(\text{T. F. D.})_{x+n+1}=l_{x+n}\left((\text{T. F. D.})_{x+n}+aP_x\right)r-d_{x+n};$$

$$\text{or } (\text{T. F. D.})_{x+n+1}=r\frac{l_{x+n}}{l_{x+n+1}}\left((\text{T. F. D.})_{x+n}+aP_x\right)-\frac{d_{x+n}}{l_{x+n+1}}.$$

It will be noticed that this formula, too, requires that, before the deposit at the end of any policy year can be calculated, the deposit at the end of the next preceding year must be known. All the other terms of the second member of the equation are known quantities.

To determine, by this formula, the amount that must be in deposit at the end of the first year on a twenty years' endowment and insurance policy for $1,000, taken out at age thirty.

In this case $x=30$ and $n=0$. The formula then becomes:

$$(\text{T. F. D.})_{x+1}=r\frac{l_x}{l_{x+1}}\Big((\text{T. F. D.})_x+a\text{P}_x\Big)-\frac{d_x}{l_{x+1}};$$

but $rv=1$; and $r=\frac{1}{v}=1.04$. From the Mortality Table we find $l_{30}=86292$, and $l_{31}=85565$. $(\text{T. F. D.})_x$ is equal to zero, because there is no deposit at the end of the year preceding the first policy year. $a\text{P}_x$, the net annual premium, is $0.03795, as previously calculated; and d_{30} by the table is 727.

Substituting these arithmetical amounts in the above formula and it becomes—

$$(\text{T. F. D.})_{31}=1.04\times\frac{86292}{85565}\times 0.03795-\frac{727}{85565};$$

or $(\text{T. F. D.})_{31}=\$0.031307$. Multiply this by 1,000, we have $31.31 for the amount of the deposit at the end of the first year. The precise figures are $31.307, which is thirty-one dollars, thirty cents, and seven mills, or $31.31.

METHOD OF CALCULATING THE NET VALUE OF A POLICY FOR FRACTIONAL PARTS OF ANY POLICY YEAR.

At the time the first premium is paid, which is at the beginning of the first policy year, the net value of the policy is the net annual premium. At the end of the first policy year the net cost of insurance will have been paid, and there must be left in the hands of the Company, in trust for the policy-holder, the requisite "Deposit." This Deposit (or "*Reserve*," as it is often called) is the net value of the policy at the end of the first policy year. At the beginning of the second policy year the net annual premium is paid, and the net value of the policy is then the "Deposit" at the end of the preceding year, plus the net annual premium just paid. The net value of the policy at the end of the second policy year is the Deposit (or "*Reserve*") for the end of that year.

In general terms, the net value of the policy, at the beginning of the n^{th} policy year, is equal to the "Deposit" at the end of the n^{th}—1 policy year, plus the net annual premium; and the net value of the policy, at the end of the n^{th} policy year, is equal to the "Deposit" at the end of n years.

This is true, because the net annual premium is sufficient, and only sufficient, when added to the "Deposit" at the end of the preceding year, to pay the net cost of insurance during the year, and provide the requisite deposit for the end of the year. Of course it is understood that net or table interest is realized for the year.

On the supposition that a policy was taken out on the 1st day of January, 1869, the net value of the policy on that day, is equal to the net annual premium just paid. On the 31st December, 1869, the net value is equal to the "Deposit" at the end of the first policy year. On the 1st day of January, 1870, the net value of the policy, just after the net annual premium is paid, is equal to the Deposit at the end of the preceding year, plus the net annual premium; and the net value on the 31st of December, 1870, will be equal to the Deposit at the end of the second policy year.

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Having in this way determined the value of this policy at the beginning and at the end of any policy year: subtract one from the other, and by this means obtain the difference between the net value on the 1st day of January and the net value on the 31st day of December of that year. Divide this difference by twelve : we will obtain the monthly difference in the net value. Assuming that the net value of a policy is greater at the beginning, than it is at the end of the policy year in question; having found the monthly difference as above, we will subtract this monthly difference from the net value at the beginning of the year, in order to find the net value of this policy on the 1st day of February of that policy year. To find the net value of the policy on the 1st day of March, we will subtract the monthly difference from the net value on the 1st day of February; and in like manner we obtain the net value of the policy at the beginning of any mont ı of the policy year, by subtracting from the net value at the beginning of the policy year, this monthly difference, multiplied by the number of months of the policy year that have expired.

On the 1st day of November, for instance, we obtain the net value by multiplying the monthly difference by ten, and subtracting the result from the net value of the policy on the 1st day of January, which day we have assumed to be, in this case, the first day of the policy year.

To obtain the net value on any day during a month, divide the monthly difference by thirty, in order to obtain the daily difference; and then use the daily difference in a manner entirely similar to that indicated above for finding the value of the policy at the beginning of any month.

Policies are taken out any day of the year, and it is usual in Life Insurance Companies to have the net valuation of all policies computed on some one day every year. The day fixed for these valuations is generally the 31st of December.

The question will then arise every year, what is the net value, on the 31st of December, of each policy in force on that day?

First determine what policy year the given policy is in at the time. Obtain its net value at the beginning of that policy year, and its net value at the end of that policy year. Take the difference between these two net values: divide this difference by twelve, in order to obtain the monthly difference in the net value; divide the monthly difference by thirty, in order to obtain the daily difference in net value. Then fix the month and day of the calendar year on which the policy was issued. The number of months and days that have, on the 31st of December, elapsed since the beginning of the policy year, will become known, and the net value of the policy on the 31st day of December can be determined by the general method above indicated.

Value at the End of a Policy Year is Sometimes Greater than the Value at the Beginning.

Owing to some peculiarity in the rate of mortality for the year, and the accumulation of interest arising from the funds on deposit, it happens at times that the net value of a policy, at the beginning of a year, will, at net interest, produce, during the year, an amount sufficient to pay the cost of insurance during the year, and provide for a "Deposit" at the end of the policy year, greater than the net value at the beginning of the year. In this case, the monthly and daily differences must be added to the net value at the beginning of the year, instead of being subtracted from it. This peculiar case does not happen in the earlier years of a policy; it is only after there is marked accumulation in the "Deposit" or net value at the end of a year, that the net value at the beginning of a year will, at net interest, produce an amount sufficient to pay the cost of insurance during the year, and leave on hand at the end of the year a "Deposit," or net value, greater than that at the beginning of the year.

These "*perturbations*" in the relative net values at the beginning and end of different years are indicated in the formula by unmistakable signs; they, in no degree, complicate the calculations, but require close observation on the part of computers to prevent mistakes.

To Find the Net Value During the Year by Using Net Value at the End of the Year.

The net value, at any time during a policy year, can be obtained with equal certainty by basing the calculations upon the Deposit or net value of the policy at the end of the policy year, instead of, as above, upon the net value at the beginning of the policy year.

Having calculated the Deposit that must be on hand at the end of the policy year, the value of the policy at the end of the first month of the policy year may be obtained by adding to the Deposit that must be on hand at the end of the year eleven twelfths of the cost of insurance during the year. At the end of the second month the net value may be obtained by adding to the Deposit that must be on hand at the end of the year ten twelfths of the cost of insurance during the year. At the end of the eleventh month one twelfth is added. At the end of the twelfth month, or end of the policy year, there is nothing to be added. The net value, and the Deposit at the end of the year, are equal quantities.

What is said above in reference to the particular case in which the Deposit or net value at the end of a policy year is greater than the net value at the beginning of the year, applies here; and, therefore, when the case occurs, the eleven twelfths of the difference between the net value at the beginning and that at the end of the year, must be subtracted from the net value or Deposit at the end of the year, in order to obtain the net value at the end of the first month of the policy year; and in like manner for other months.

It is assumed in both of the methods for calculating the net value of a policy during the policy year, that the variation in value is proportional to the time; and that each month has thirty days.

VALUATION TABLES.

The net values of different kinds of policies on the 31st of December, in each policy year, have been calculated and arranged in VALUATION TABLES convenient for use. Without the aid of these "Valuation Tables," the work of computing the net value of every policy in all the Companies would be an almost impracticable labor. Even with the aid of "Valuation Tables," the work is enormous, as may be readily comprehended from the fact that one single Company has more than sixty thousand policies in force.

"THE PROBLEM THAT STANDS AT THE THRESHOLD OF LIFE INSURANCE," AND SOMETHING THAT IS WITHIN THE THRESHOLD.

It has been previously stated, that, by the payment at any age x of the *net single premium* due to that age, the insured places in the hands of a Life Insurance Company enough money to pay the cost of insurance during the first policy year, and leave in the hands of the Company on deposit, at the end of that year, an amount equal to the net single premium at the age $x+1$. To effect this, the Company must realize net or table interest upon the net single premium, paid at the age x. And in like manner the net single premium, on deposit at the age $x+1$, will pay for the cost of insurance during the second policy year, and leave in deposit with the Company, at the end of the second policy year, an amount equal to the net single premium at the age $x+2$, and so on to the end.

By the payment, at any age x, of the *net annual premium* due to that age, the insured places in the hands of the Company money enough to pay the cost of insurance during the first policy year, and leave in the hands of the Company on deposit, at the end of that year, an amount sufficient to purchase, at that time, a life series of annual premiums, each of which is equal to the difference between the net annual premium due to the age x, and that due to the age $x+1$. And in like manner, for the following years, to the end.

Each of these methods requires a *deposit* at the end of each policy year; and it has been seen that this deposit increases every year for each outstanding policy. And in time this deposit, in Companies that have a large number of policies that have been in force for years, amounts to enormous sums of money. The safe and advantageous investment of this Trust Fund Deposit is the great "*question of finance*" in Life Insurance. "*The proper handling and control of these enormous sums of money, belonging to other people, is the onerous trust imposed upon the officers and directors of Life Insurance Companies.*"

The term "*Cost of Insurance*," used in this connection, is not the actual value of the risk during any year upon an amount equal to the amouut insured; because, from the moment the insured pays his first premium, the Company has in its hands, of the policy-holders' money, enough to pay the cost of insurance during the year, and leave the requisite deposit at the end of the year. Therefore, from the time the first payment is made, the actual risk incurred by the Company for the year is not upon the amount of the policy, *but it is this amount, less the deposit at the end of the year.* It has been previously seen that we first calculate the value of the risk on the policy (of $1), and then calculate the cost of insurance (in these two cases of payments by net single premium or by net annual premium), by finding the amount at risk during the year, and then using the following proportion, viz: "The amount of the policy is to the amount at risk, as the value of the risk during the year on an amount equal to the policy, is to the value of the risk on the amount actually at risk." The latter is what is generally called "Cost of Insurance."

It has been seen that the net value that will, at age x, insure one dollar for the first year, is represented by $v\frac{d_x}{l_x}$, and that at the same age x, the net value at that time that will insure one dollar, to be paid at the end of the second year, *provided* the insured dies during the second year, is represented by $v^2\frac{d_{x+1}}{l_x}$. Similar expressions will give the means for determining at the age x, what it is then worth to insure one dollar, to be paid to the heirs of the insured at the end of the n^{th} year, *provided* the insured dies during that year. The method of calculating the arithmetical value of v, at any given rate of interest, has been already very fully explained. Of course, when v is determined, the second power, third power, or the n^{th} power of v, is easily calculated; and the Mortality Table gives the number of deaths during any year, and the number living at the age x. It is, therefore, seen that it is not only very easy (*after a Mortality Table is furnished, and a rate of net interest is fixed upon*) to calculate what it will cost, at any age x, to insure one dollar, to be paid to the heirs of the insured at the end of the first year, in case he dies during the year; but it is easy to calculate what it will cost at age x to insure one dollar, to be paid at the end of n years, *provided* the insured dies during the n^{th} year. This subject is treated at some length in the beginning of these Notes, and an arithmetical illustration is given on page 19. The subject has been again referred to here, in order to remind the reader that "*the rate of premium that must be charged, in order to carry out an insurance contract, is the problem which stands at the threshold of Life Assurance;*" and in addition, to call his attention once more to that peculiar element in Life Insurance which is styled "*Reserve*" by most writers on this subject, and in these Notes is called Trust Fund Deposit, and convince him that, after passing the threshold, there are some peculiar things connected with this business, as at present conducted, that demand special and close examination.

There is a direct and simple method of insurance (indicated by the expression $v \frac{d_x}{l_x}$, as seen above), by which the Trust Fund Deposit is entirely avoided; the complicated accounts inherent in the "*System of Deposits*" are done away with; and Life Insurance, year by year, made simpler than by the payment of net annual premiums, or net single premiums. But there are serious drawbacks in this case, provided a man desires to keep his life always insured to the table limit of ninety-nine years. But it is not so clear that those persons who desire to insure their lives, year by year, or for a short period of years, would not find it advantageous to pay the amount $v\frac{d_x}{l_x}$ each year: or if he desired to insure for five years, and avoid medical examinations after the first examination for admission, let him pay in advance for the whole five years; for the first year $v\frac{d_x}{l_x}$; for the second year $v^2\frac{d_{x+1}}{l_x}$; for the third year $v^3\frac{d_{x+2}}{l_x}$; for the fourth year $v^4\frac{d_{x+3}}{l_x}$; and for the fifth year $v^5\frac{d_{x+4}}{l_x}$.

The sum of these yearly amounts, in advance, will pay for an insurance for five years. For a limited number of years it might be assumed that a higher rate of interest could be realized, say the investments were made in safe seven per cent. bonds, or bonds and mortgages on real estate worth two or three times the amount of the mortgage. In this case of interest at seven per cent., v becomes equal to 100 divided by 107, or $0.934579. At age thirty, it will cost to insure one dollar for one year, $v\frac{d_x}{l_x}=\$0.934579\times\frac{727}{86292}=$ $0.007873. This will pay for the insurance of one dollar for one year at the age thirty, assuming the Actuaries' Table of Mortality, and interest at the rate of seven per cent. Multiply by 1,000, and we have $7.87; this amount will pay for $1,000 insurance for one year at age thirty. The annual premium at same age, ACTUALLY CHARGED by first-class Life Insurance Companies, is about $23.00—varying slightly in the different Companies. But it must be remembered that the net annual premium, besides paying for Cost of Insurance, must provide for the requisite deposit at the end of the year; and THE PREMIUM ACTUALLY CHARGED, must always be greater than the net annual premium, by an amount sufficient to pay ALL THE EXPENSES connected, DIRECTLY AND INDIRECTLY, with the conduct of the business, other than the cost of insurance or losses by death properly chargeable to this policy.

NOTE.—The manner of constructing the Tables, and the arithmetical rules for working out the formulas, and the Tables used in making the net calculations, have, for convenient reference, been taken out of Part First and placed at the end of the book.

Mr. Gladstone, the Chancellor of the Exchequer, in a speech delivered March 7th, 1864, in the British House of Commons, said:

"Consider for a moment the peculiar nature of Life Assurance. This is a business that presents the direct converse of ordinary commercial business. Ordinary commercial business, if legitimate, begins with a considerable investment of capital, and the profits follow, perhaps, at a considerable distance. But here, on the contrary, you begin with receiving largely, and your liabilities are postponed to a distant date. Now I dare say there are not many members of this House who know to what an extraordinary extent this is true, and, therefore, to what an extraordinary extent the public are dependent on the prudence, the high honor, and the character of those concerned in the management of these institutions. When an institution of this kind is founded, so far from having difficulties at the outstart, that is the time of its glory and enjoyment. The money comes rolling in, and the claims are at a distance almost beyond the horizon."

Extract from the report of a Committee of the British Parliament in 1853. [The party under examination was the Actuary of the "Royal Exchange Assurance Office."]

Question: "Do you think there is any thing peculiar in the character of Life Assurance business which would justify the Legislature in interfering with it in a way different from other business?"

Answer: "Yes, both on account of the long period over which the contracts extend, and especially for this reason: that Life Assurance offices are now taking to make up their accounts on principles that would be scouted from any other department of commercial enterprise."

Question: "Will you explain what principle you mean?"

Answer: "The practice of anticipating future profits and treating them as assets. Allow me to suppose the case of a bank making up its accounts: it owes to its depositors £1,000,000; it has on hand £900,000; it puts down as an additional item of assets, profits, we will say at the rate of £10,000 a year, valued at twenty years' purchase; by that means it makes its assets £1,100,000, against £1,000,000, and the result is stated to be a surplus of £100,000. That principle would never be adopted in a bank, and I think it ought not to be adopted in an Assurance Company."

Question: "But, does it exist in Assurance Companies?"

Answer: "It is done."

Question: "Is it done by Assurance Companies generally, or only in particular cases?"

Answer: "It is in considerable use, and the practice is extending."

"There can hardly be a happier set of capitalists on earth than one which has obtained a right, by perpetual charter, to insure lives, receiving from the proceeds, first legal interest semi-annually on stock as a sure thing, and secondly, twenty per cent. of what are called profits, that is premiums proving to be surplus, if it can once secure a flourishing business."

Elizur Wright (1862).

PART SECOND.

PRACTICAL LIFE INSURANCE.

Expenses—Loading—Surplus.

In order to pay the Expenses of conducting the business of "Practical Life Insurance," it is necessary that additional means should be provided over and above the *net* annual premiums; the latter being enough, and only enough, when regularly compounded yearly at four per cent., to pay the Cost of Insurance, and furnish the requisite Deposit or "*Reserve*."

It is usual to add to the net annual premium from twenty to thirty per cent., or even more, for the purpose of defraying expenses. This addition to the net annual premium is technically called Loading.

The Loading may, and often does, more than pay expenses.

The interest actually received is nearly always more than the net interest assumed in the table calculations.

And the actual mortality, particularly in the earlier years of a Company, is, in practice, generally less than that given in the table.

From each of the three above-named sources Surplus may be obtained. By Surplus is here meant *money*, or its equivalent, in excess of what is required to pay losses by death during the year, to form the "*Deposit*" for the policy at the end of the year, and pay all expenses. The surplus, in *purely* Mutual Companies, belongs to the policy-holders. In the *purely* Stock Companies all the surplus goes to the share-holders. The *mixed* Companies are those Stock Companies that give some portion of the surplus to the policy-holders.

In order to investigate the nature of practical Life Insurance business for one year, let us suppose that the Cost of

Insurance, and all expenses of the previous year, have been paid, and that the Company had on hand, at the close of the previous year, the requisite Deposit for each and all of its outstanding policies. We will, for the present, suppose that the Surplus of the previous year had been distributed to its respective owners.

At the beginning of the year, the business of which, we are now investigating, each policy-holder pays his *full* annual premium; that is, the Loading is included. There is then in the hands of the Company, on account and to the *credit* of each policy, the two amounts, viz: the Deposit at the end of the preceding year, and the full annual premium.

These sums are both invested at the best, safe rate of interest; and out of these two amounts, thus increased by interest, actually received during the year, the "expenses" for the year, properly chargeable to each policy, must be paid. The Cost of Insurance, or proportion of losses by death during the year, properly chargeable to each policy, must be paid. And the requisite Deposit at the end of the year for each policy, must be provided and set apart, or securely invested for the policy-holder at the net, or table rate of interest at least. If there is anything left on account of each policy, it is Surplus produced by the policy.

(1.) Let $(\text{T. F. D.})_{x+n}\ (1+r')$ *represent the* DEPOSIT *at the end of the* n^{th} *year from the date of the policy,* INCREASED *by the interest actually received upon it during the year now in question; i. e.* $(n^{th}+1)$. r' *represents* THE RATE OF INTEREST DIVIDED BY 100.

(2.) *Let* $(P_x - e)\ (1+r')$ *represent that the expenses chargeable to this policy, for the year, are taken out of the full annual premium, and the remainder of the premium is increased by the interest actually received upon it during the year. In this,* **P** *is the* GROSS OR FULL PREMIUM; e *represents* EXPENSES; *and* r', *as above, represents* THE RATE OF INTEREST DIVIDED BY 100.

(3.) *Let* $\frac{d'_{x+n}}{l'_{x+n}}\left(1-(\text{T. F. D.})_{x+n+1}\right)$ *represent the actual* "COST OF INSURANCE" *during the year;* d' *represents the actual number of deaths during the year;* l' *represents the number actually living at the beginning of the year.*

(4.) *Let* $(T.\ F.\ D.)_{x+n+1}$ *represent the* DEPOSIT at the. end of the $n^{th}+1$ year of the policy.

Add expression (2) to expression (1); subtract expressions (3) and (4) from the sum thus produced. If there is anything left, it is SURPLUS produced by this policy.

Combining the expressions (1), (2), (3), and (4), as above indicated, we have the equation—

$$(T.\ F.\ D.)_{x+n}(1+r')+(P_x-e)(1+r')-\frac{d'_{x+n}}{l'_{x+n}}\left(1-(T.\ F.\ D)_{x+n+1}\right)-(T.\ F.\ D.)_{x+n+1}=\text{SURPLUS PRODUCED BY THIS POLICY DURING THE } n^{th}+1 \text{ YEAR.}$$

When the SURPLUS arising from the funds of each policy is obtained as above, and is distributed in accordance with the principle expressed in this equation, it is said to be divided upon the "CONTRIBUTION PLAN."

EXPLANATION OF THE TERMS CONTAINED IN THE ABOVE EQUATION.

Notwithstanding the rather formidable appearance of the signs and symbols in the above equation, it is really very simple and plain, when expressed in the ordinary words of our language.

The first term expresses only that the DEPOSIT on hand, at the end of the n^{th} year, HAS BEEN INCREASED by the amount it actually produced when placed at the best safe rate of interest during the $n^{th}+1$ year. r' is not the rate of interest, but is a quantity which, when multiplied by the Deposit, will give us the amount actually received from interest during the year. Suppose this rate of interest had been seven per cent., then r' is equal to $\frac{7}{100}$. The second term expresses that from P, the full annual premium, we subtract the EXPENSES actually incurred and properly chargeable to this policy during the $n^{th}+1$ year, and place the remainder, which is $(P-e)$, at the best safe rate of interest; and add the interest so obtained, to what was left of the full annual premium after the EXPENSES had been deducted.

The third term is the actual Cost of Insurance during the $n^{th}+1$ year, properly chargeable to this policy. In which l'_{x+n} represents the number actually living at the end of n years from the date of the policy; that is, at the beginning of the $n^{th}+1$ year; and d'_{x+n} represents the number that actually die during the year between age $x+n$ and the age $x+n+1$. The second factor of this third term is the amount the Company has at risk during the year.

The fourth term is the requisite Deposit that must be on hand at the end of the $n^{th}+1$ year from the date of the policy.

Variations from the Table Rate of Mortality.

We have already explained how to calculate the requisite Deposit. The Amount at Risk, too, is a specific sum, and is equal to the amount of the policy, minus the Deposit for the end of the year in question. But the fraction obtained by dividing the number of actual deaths during the year by the actual number living at the beginning of the year, occasionally varies somewhat, and may vary greatly, from the general law of mortality amongst large numbers of mankind. This is particularly liable to be the case when the Company has but a limited number of policy-holders; and the variation would be very marked, whether the whole number of policy-holders in the Company was large or small, in case sickly or diseased persons were taken by the Company. It is worthy of notice, too, that if the number of deaths, in any one year, should prove to be remarkably small, it is not safe to assume that, because the losses by death, in that year, are greatly less than those called for by the Table of Mortality, that the difference is clear gain, and can be disposed of as "Surplus," and distributed at the end of the year; because the variation from the number of deaths called for by the Table of Mortality, will probably soon vary on the other side; and, when the losses are materially greater, by death during the year, than called for by the table, these losses have to be paid, and that promptly. It is, therefore, well to have something in hand to pay with. The Company cannot touch its Deposit, because that money *does not belong to the Company*. It must not only be held for its proper owner, but it must always be earning interest for its owner; and this interest must be regularly compounded every year. Purely Mutual Companies, if well conducted, always keep back a portion of their Surplus. One of these Companies, in its last official report, has kept back, and no doubt wisely, nearly two and one quarter millions of its Surplus. It is a large Company, has large risks, and ought to endeavor by all prudent means to meet its risks.

Effect of Allowing a Policy for a Large Amount to be Grouped With Those of Small Amount.

Again referring to the term in the above equation that expresses the Cost of Insurance, properly chargeable to the policy for the year, it will be noticed, that although the Amount at Risk on the policy is a factor in that term, and to this extent, the amount of the policy, however large, has been fully considered and allowed for, still, if a one hundred thousand dollar policy had been, by the Company, grouped in the accounts with a number of policies of smaller amount, but of the same age, say with ninety-nine others of one thousand dollars each, the death of the single individual in this case would be a greater loss to the Company than that of the whole other ninety-nine policy-holders. These things, and many others of a similar nature, have to be closely watched and strictly attended to by officers of the greatest skill and good judgment. And whilst merely theoretical information is not enough to qualify an officer to conduct this or any other business successfully, it may be set down as *certain* that no man can conduct Life Insurance safely and properly unless he knows something of the principles upon which it is founded.

Further Allusion to Loading and Expenses.

Let us now consider the Loading which has been added to the net annual premium, and the Expense which this Loading is intended to provide for. In the first place, it may be remarked that the "Expenses," properly chargeable to a policy, are not necessarily the same proportion of the annual premium in different cases. At the end of the year, although it may require some labor to adjust with precision the Expense account for each separate policy, or each distinctive set of policies, this should be attempted, and substantial equity in this respect can always be attained.

If the amount of Expenses, and the interest that will be realized in the most unfavorable year, during the continuance of a policy, could be accurately ascertained in advance, the Loading would, even in that case, have to be enough to meet the worst year.

Certainty of the payment of his policy at maturity is what every man wants when he insures his life. The quality of the article he purchases is the first and the greatest consideration with him. It must be certain that the Life Insurance Company has charged enough to enable it to pass safely through the worst years that can reasonably be expected to occur during the period of the contract between the Company and the insured, which is generally supposed to be for a lifetime.

SURPLUS.

In favorable, or even in ordinary years, the Loading, and the interest on the funds of the Company (because of their realizing usually a higher rate than that called for by the table calculations), will produce a "Surplus" on each policy at the end of the business of a year. This Surplus arises from previous *over-payment* in advance, demanded by the Company, in order to make it *certain* that the Company will be self-sustaining in the worst year that may occur in a lifetime. The Surplus distributed to policy-holders is merely a return to them of that part of the premium they paid at the beginning of the year, which, at the end of the year, is found not to have been required during the year, either in effecting the insurance, providing the means (the Deposit) for paying the policy at maturity, or in paying "EXPENSES."

Accounts to be Kept with Each Policy.

It must always be held in mind, that whilst in Life Insurance there are peculiar and mandatory arithmetical laws by which particular money values are computed—in addition, and after these values are accurately determined—practical Life Insurance becomes like all other business which involves the handling and control of vast amounts of money. Good judgment, great industry, the strictest integrity, and sound practical business sense on the part of those intrusted with the conduct of Life Insurance, are all absolutely essential to successful management.

No prudent man will ever attempt to control or conduct any important business without making some kind of estimate in advance. The Mortality Table furnishes the means for making certain estimates with an accuracy that is not usually found in ordinary business. But, in regard to the "Expenses" that will be incurred, or the rate of interest that will be realized on these investments, or the bad investments that may be made, or whether some of its officers may not prove to be dishonest, or what the Company will make or lose, and a variety of highly important questions of this nature, cannot be settled by estimates made beforehand by Life Insurance Companies any more definitely than similar estimates can be made in any other business. Nevertheless, these estimates of practical results ought always to be made in advance; but it will be a sad mistake to assume, at the end of a year, that the estimates made at the beginning of the year give, necessarily, an accurate statement of the real condition of the business. At the end of the year the estimates must be changed to fit the facts.

The question of EXPENSES in Life Insurance, as in every other business, is a matter of vital importance, and ought to receive the closest attention, not only from those who manage the business, but from those who keep the accounts. It is not proposed here to intimate even how the accounts of a Life Insurance Company ought to be kept; but only to allude to some of the main practical, ordinary matters, that are just as essential in Life Insurance as they are in all other kinds of business. When at the end of a year the EXPENSE account is made up, and the amount properly chargeable to each policy has been determined, e, in the equation, becomes known; r' is the one hundredth part of the average rate of interest actually received by the Company during the year upon its aggregate investments. The manner of calculating the net annual premium has already been explained. Add to this the Loading that has been fixed upon, and P, in the preceding equation, becomes known. The manner of calculating the DEPOSIT at both the beginning and the end of the year, and the Cost of Insurance during the year, has been previously given. There is nothing further needed, in order to make up the accounts of each separate policy for the year, after the general accounts of the year's business are made out. It must be noticed, however, that in case there are any items in the general accounts, involving either gain or loss, that do not enter, in some shape, into the separate policy accounts, the aggregate surplus, arising from the separate policies, will not be equal to the surplus shown by the general accounts to be actually on hand. It no doubt happens often that certain gains or losses during the year are not carried into the separate policy accounts on the books of a Company. In this case, when the amount of surplus produced by a policy is calculated by the above equation, the actual surplus for the policy will be obtained by the following proportion, viz: the total surplus obtained by adding together that of all the separate policies is to the actual surplus, as shown by the general accounts, as the surplus for each particular policy is to the amount of surplus to be credited to the policy in the general accounts.

In case this account includes everything not included in the other terms of the equation for obtaining the surplus, there will be no occasion for using the proportion just referred to; because, in this case, the sum obtained by adding together the surplus of all the separate policies will be just equal to the surplus shown by the general accounts.

The subject of accounts in Life Insurance Companies will never be definitely settled until the book-keepers and accountants clearly understand the theory and principles upon which Life Insurance is founded. It is safe to say, that if any money account is kept with a policy at all, it ought to be made exactly correct.

Example Illustrating the Calculation of Surplus.

The following arithmetical example is given merely in illustration of the method of making up the accounts of a policy for any year, and determining the surplus:

It is assumed that an ordinary whole life policy for one thousand dollars, taken out at age forty-two, is in its 10th year. The net annual premium, as previously calculated, is $25.55; take the loading to be 33⅓ per cent. of this; then the full annual premium is $34.05. To make out the account of this policy during the 10th year, we will assume that the expenses properly chargeable to it during the year, are, at the end of the year, found to be twenty per cent. of the full or gross annual premium; that everything to the credit of this policy at the end of the preceding year, except the Deposit, had been distributed to its owner or owners; that the rate of interest actually realized by the Company on its aggregate investments during the year, was seven per cent.; and that the mortality amongst the insured during the year, was that called for by the table.

The first thing to be done, is to calculate the DEPOSIT for the end of the preceding year:

$x=42$, $n=10$. Then $(T. F. D.)_{x+n-1}$, or $(T. F. D.)_{51}=1-\frac{A_{51}}{A_{42}}$.

$$\text{But } A_{51}=\frac{N_{51}}{D_{51}}=\frac{121983.7}{9255.77}=13.179206;$$

$$\text{and } A_{42}=\frac{N_{42}}{D_{42}}=\frac{231671.7}{14830.58}=15.621223;$$

$$\text{therefore } \frac{A_{51}}{A_{42}}=\frac{13.179206}{15.621223}=\$0.843673;$$

$$\text{and } 1-\frac{A_{51}}{A_{42}}=\$1-\$0.843673=\$0.156326;$$

which is the DEPOSIT for an ordinary whole life policy for one dollar at the end of the 9th year, the policy having been taken out at age forty-two. In a precisely similar manner, we calculate the DEPOSIT for this policy at the end of the 10th year, only noting that, in this case, the formula is $(T. F. D.)_{x+n}$, or $(T. F. D.)_{52}=1-\frac{A_{52}}{A_{42}}$. The Deposit for the end of the 10th year will be found to be \$0.175155.

The Amount at Risk during the tenth year on this policy of one dollar, was \$1 — \$0.175155 = \$0.824844. The mortality amongst the insured during the year having been assumed to have conformed to that expressed in the table, we find from the table that $\frac{1,156}{68,409}$ is the fraction that expresses the rate at which the insured persons died during the year. This fraction, multiplied by the Amount at Risk during the year, gives \$0.01393 as the amount properly chargeable to this policy of one dollar, during the year, for Cost of Insurance.

Multiply each of the above Deposits, and the Cost of Insurance by 1,000, and we have the Deposit at the end of the ninth year equal to \$156.33; the Deposit at the end of the tenth year equal to $175.16; and the Cost of Insurance during the tenth year equal to $13.93 on an ordinary whole life policy for $1,000 taken out at age forty-two.

We are now Ready to Make out the Account of this Policy for the Tenth Policy Year.

The Deposit at the end of the ninth or beginning of the tenth year was $156.33. The full or gross annual premium is $34.05, twenty per cent. of which is deducted for expenses; leaving on hand at the beginning of the year $27.24 of this premium. Add this to the Deposit on hand at the end of the preceding, or beginning of the present year, and we have to the credit of the policy $183.57, after expenses for the year have been provided for.

Add to this, interest for one year, at seven per cent., which is $12.85, and we have to the credit of the policy, in the account for the tenth year, $196.42. Deduct from this the Cost of Insurance, $13.93, and we have $182.49. Take out the Deposit that the Company must have on hand for this policy at the end of the tenth year, $175.16, and there is $7.33 Surplus, *which is about twenty-one per cent. of the full annual premium paid.*

But in case the expenses for the year, and the mortality amongst the insured, had been greater than that assumed in this example, and the interest had been less, this surplus would have been diminished. On the other hand, had the variation in expenses, mortality, and interest, been the opposite of the above, the Surplus would have been greater. Certainty of payment of his policy at maturity is what every policy-holder wants; and it is fair to suppose that both he and his heirs would prefer to have the whole amount paid in money. We have seen above, that, with a Loading of $33\frac{1}{3}$ per cent. on the net annual premium, there was, at the end of the year, a Surplus of $7.33: *no great margin, when the question is that of the prompt and certain payment at maturity of a policy of* ONE THOUSAND DOLLARS, *more especially when the Surplus or* OVER-PAYMENT MADE AT THE BEGINNING OF THE YEAR, IN ORDER TO MAKE THE PAYMENT OF THE POLICY SAFE, IS RETURNED TO THE POLICY-HOLDER AT THE END OF THE YEAR.

When the Surplus belonging to the policy-holder is not distributed, but remains in the hands of the Company to the credit of the policy that produced it, it ought to be invested for the holder of the policy. When the Surplus has all been distributed, the true value of the policy at the end of any year, and before the payment of the next annual premium, is the DEPOSIT; but when it has not been distributed, the true value of the policy is the *Deposit*, *plus any Surplus there may be in the hands of the Company to the credit of the policy.*

ADDITIONAL INSURANCE PURCHASED WITH THE SURPLUS.

When the Surplus is distributed to the policy-holders, it may be used in part payment of the next annual premium, or, at the option of the policy-holder, it may be applied to the purchase of additional full-paid insurance. The latter would progressively increase the amount of the policy; the former would result in a progressively diminishing annual premium.

When the amount of Surplus to be returned has been determined, the amount of full-paid insurance that this Surplus will purchase at that age is calculated by first finding the net single premium that will insure one dollar at that age. This we have agreed to represent by sP_x; and, as previously shown, $sP_x = 1 - (1 - v)\,A_x$, and its value is calculated by replacing x by the age in question; and obtaining the values of N_x and D_x opposite that age in the table; this numerator and denominator gives us a fraction which is the value of A at that age. The numerical value of v has previously been determined; and we can, therefore, convert the second member of the above equation into its proper arithmetical value. This will give THE NET SINGLE PREMIUM sP_x, WHICH, AT THAT AGE, WILL INSURE ONE DOLLAR. Of course the surplus, used as a net single premium at that age to purchase additional full-paid insurance, will buy a proportional amount.

Take, for example, age thirty. Suppose that the surplus is $15.36, and the policy-holder desires to purchase with this an addition to his policy, instead of using it in part payment of his next annual premium.

First find what net single premium will, at that age, insure one dollar. The equation becomes $sP_{30}=1-(1-v)\frac{N_{30}}{D_{30}}$. From the table we find $N_{30}=479.951.6$; and $D_{30}=26.605.37$. As before calculated, we have $v=\$0.961538$. Therefore $(1-v)=\$0.038462$. Multiply this by N_{30} and divide the product by D_{30}. Subtract the result from unity, and we have $0.306158. This is the net single premium that will, at age thirty, insure one dollar, to be paid to the heirs of the insured at the end of the year in which he may die. And the question in arithmetic is this: if $0.306158 will insure one dollar, how much will $15.36 insure? The answer is, $50.17; and this is the amount of addition to the policy that the surplus named will purchase. This additional insurance is full paid, and the $50.17, in this case, is called by insurance writers the "REVERSIONARY VALUE" of the surplus, $15.36.

Any proceeds that may in the future arise from interest on this $15.36, in excess of the four per cent. necessary to pay the Cost of Insurance and provide the requisite Deposit, will be additional surplus, and may be used as it accrues in purchasing additional full-paid insurance. It is estimated that a whole life policy for $10,000, taken out at age thirty, by the payment of an annual premium of $230.20 every year for twenty-three years, will, in that time, amount to $17,906.50, if the surplus arising from over-payments is applied to the purchase of additional full-paid insurance; and the surplus arising from interest over four per cent. upon the net single premium, paid for the additional insurance, is applied yearly to increase the policy. Moreover, at the end of twenty-three years, the policy will have become virtually full paid; because it is estimated that the return of surplus, after that time, will exceed the annual premium, and there will be no further payments required. On the contrary, as the return of surplus now exceeds the annual premium, the policy-holder may receive an income, or REAL DIVIDEND, from his policy; or by leaving the money in the hands of the Company, his policy will go on increasing, from year to year, without any further payment of premiums until it matures at his death.

The peculiar and attractive feature in CASH Companies that return the surplus to the owner of the policy, either in cash, diminishing by that amount the annual premium required, or by purchasing with the surplus additional full-paid insurance, is virtually unknown in NOTE Companies or in the STRICTLY PROPRIETARY Companies. In the latter all the surplus goes direct to the share-holders; and the insured pays a given amount every year, without addition or diminution of his annual premiums or of his policy. In the NOTE

Companies the surplus to policy-holders generally consists, in great part, of their own notes; and there is, in most cases, no diminution of the amount of premium required from year to year; but there is often a diminution in the amount of the policy, arising from an accumulation of notes of the policy-holder, which must be deducted from the policy at maturity. It is believed that policy-holders who have been insured in NOTE Companies for a period of years, are quite sure to realize the fact that the burden upon them is getting heavier instead of lighter, and that IT IS ONLY THOSE WHO DIE SOON AFTER GETTING INSURED THAT CAN POSSIBLY GAIN ANY ADVANTAGE FROM THE SYSTEM OF INSURING UPON CREDIT, RATHER THAN FOR CASH.

Comparison Between a Cash Company that Returns the Surplus to the Policy-holders and a Note Company that Retains Twenty Per Cent. of the Surplus.

In illustration of some of the features of the system of Life Insurance on credit, and the effect of appropriating to the shareholders twenty per cent. of the surplus of the policy-holders, we will, for illustration, suppose a Company has $100,000 full-paid capital stock. It receives one-third of its premiums in notes, or makes a loan of that amount to the policy-holders; and by its charter has secured the right to twenty per cent. of the surplus in CASH, whilst eighty per cent. of the surplus is returned to the policy-holder by canceling on the books that much of a LOAN the Company is supposed to have made to him. The policy is for whole life $10,000, and taken out at age thirty. The annual premium in this Company is $233.00. The annual premium on a similar policy in the All-cash Company, with which the loan is proposed to be compared, is $230.20.

At first sight it might appear that the Loan Company has the advantage in price, because its premium is only $2.80 more than that in the All-cash Company, and it loans the policy-holder one-third of the money, and only charges him six per cent. interest on the loan. But let us take up the accounts of this policy in the Loan Company, and see how it will stand at the end of the first year, and then take it year by year to the limit of the table; that is, up to, and including, ninety-nine years of age.

By the formula previously deduced, let us calculate the DEPOSIT that must be on hand at the end of the first year in order to make the payment of the policy IN MONEY secure. It will be found to be $93.10.

By the Table of Mortality we use, which is the Actuaries', and four per cent. net interest, the cost of insurance for the year will be found to be $83.46. But it is claimed by some Companies that "American Experience" among insured lives is more favorable than the above; and that by this table, with net interest at four and a half per cent., twenty per cent. of this Cost of Insurance may be abated. We will, for the purposes of the present examination and comparison, assume that the Cost of Insurance, in practice, is only four fifths of what is called for by the calculations, based upon the Actuaries' table, four per cent. We therefore take this Cost of Insurance as being $66.77, instead of $83.46.

It is assumed that the expenses will be twenty per cent. of the premium, or $46.60 per year. On account of this policy, then, for the first year, there must be paid for Cost of Insurance, $66.77; for expenses, $46.60; and a DEPOSIT of $93.10 must be on hand at the end of the year. This makes the LIABILITIES of the Company on this policy, for the first year, amount to $206.47.

Let us now see how the other side of the account of this policy stands. The annual premium is $233.00; but $77.67 of this is LOAN. Cash, $155.33. Six per cent. cash in advance on the loan is $4.66. Add this to the cash part of the premium, and we have $160.00 CASH received by the Company at the beginning of the year. Place this at seven per cent., which is the rate we have assumed the Company will realize, and regularly compound during the existence of the contract, or for sixty-nine years.

The interest upon $160.00 for one year would, at seven per cent., be $11.20; hence, the principal and interest amounts to $171.20, with which to meet the liabilities for the year, viz: $206.47. This shows a DEFICIENCY OF CASH at the end of the first year amounting to $35.37. But by counting the LOAN as an asset, and adding it to the CASH, we can make out a SURPLUS, and make *so-called* DIVIDENDS to policy-holders, and REAL DIVIDENDS to share-holders. The loan is $77.67; add this to the cash, $171.20, and we have $248.87 with which to meet $206.47 of liabilities. This makes the Surplus $42.40, twenty per cent. of which, or $8.48 IN CASH, GOES TO THE SHARE-HOLDERS; but as there is no cash in this *so-called* SURPLUS, the CASH TO PAY THE SHARE-HOLDERS MUST BE OBTAINED FROM SOME OTHER SOURCE. We have seen that the calculations call for $93.10 in DEPOSIT; but after paying the COST OF INSURANCE and EXPENSES, there was left only $57.83 in cash. That the share-holders may receive their twenty per cent. in cash, the amount $8.48 must be taken from the ALREADY INADEQUATE DEPOSIT, and its place there be supplied by an equal amount taken from the SURPLUS, all of which is LOAN. *The dividend to the policy-holder is made by canceling on the books an amount equal to eighty per cent. of the* SURPLUS LOAN, viz: $33.92, WHILST THE SHARE-HOLDER TAKES INTO HIS PRIVATE POSSESSION AND OWNERSHIP $8.48 IN CASH; *and the Company has in Deposit, at the end of the first year, but* $49.35 *in money, instead of the* $93.10 *it should have in order to secure the payment of the policy in money at maturity.*

At the beginning of the second year there is in the hands of the Company, to the credit of this policy, this fragment of the Deposit that the calculations call for on a cash basis. The annual premium is paid as before. Deduct twenty per cent. of the premium for expenses; place the money that is left in the hands of the Company at seven per cent.; calculate the Deposit for the end of the second year; calculate the Cost of Insurance during the year; find the Surplus as before; take twenty per cent. of the Surplus, and place it in the Deposit; take an equal amount of cash out of the Deposit, hand it to the share-holders, and give eighty per cent. of the Surplus, which is all loan, to the policy-holder, for his DIVIDEND on the business of the second year.

The calculations have been made following the accounts of this Company, year by year, to, and including, age ninety-nine. They are not given here, because it is believed that the tables and formula will enable any tolerably fair arithmetical computer to make them for himself. It is proposed here to give only a brief summary of the main facts developed by the calculations. In the first place, IT WAS FIFTY YEARS BEFORE THE INSURED COULD WORK HIS POLICY OUT OF DEBT. At the age of EIGHTY he was for the first time really insured for $10,000, and he had by this time paid exactly $8,000 IN CASH to the Company, and he will have to pay $160.00 CASH the next year. He has now reached, however, the point at which his annual premiums may be reduced by a return of real Surplus; or, if he prefers to continue to pay the $160.00 annually, his policy may now begin to increase.

Let us look at the policy in the All-cash Company. We will not refer to the detail calculations of the first year, because there are no complications in this case—IT IS ALL CASH; no cutting up of amounts and taking NOTES or LOANS from one place, to replace CASH REMOVED FROM ANOTHER. We find that in the Cash Company the policy had become virtually paid up by the end of the thirty-eighth year, because the return of annual surplus in cash had by that time become equal to the annual premium. The insured had paid, in all, $4,077.94, and was done paying; but his return of surplus will continue to increase; and so far from having any thing to pay on his policy after the thirty-eighth year, he will receive an annual CASH DIVIDEND from it, or INCOME. Not one of those "*so-called*" DIVIDENDS, by which I give you $100 and my *note* for $30 now; and at the end of one year I give you another $100 in cash and my note for $30, and you hand me back the first note for $30 and call that a "DIVIDEND" of thirty per cent., and make me believe that I have made a thirty per cent. investment, and am getting rich, when, in fact, there has been no real use for the NOTE or LOAN, except to return it to me, or give it to my heirs as money in case I die. The policy-holder in the Cash Company above described is done paying at the end of thirty-eight years for the policy he bought, and is now receiving a clear income from the surplus price he paid for it. We have seen the man in the Loan Company reach the fiftieth year. He had paid $8,000 CASH, and was still paying $160 per year IN MONEY; but is now for the first time relieved of the necessity for accepting LOANS from the Company.

Let us see how the accounts of the old man's policy will stand at the end of the sixty-ninth year, on the supposition that his surplus had been returned to him in part payment of his cash annual premiums since he passed eighty years. We find him at the beginning of the sixty-ninth year only required to pay, in cash, $79.10. This, and the return surplus he was entitled to, makes up the cash part of his regular annual premium, $155.33. We find his Deposit at the end of this year to be $9,445.70. If to this we add the NET ANNUAL PREMIUM, *actually due to the age thirty, at which he entered the Company*, $169.70, instead of the cash part of the annual premium he has been paying, we find the sum in the hands of the Company, to the credit of this policy-holder, the day he is ninety-nine years old, will be $9,615.40; and this, at four per cent., will, by the end of the year, produce $384.61 interest, which, added to the above sum, makes $10,000.

We accompanied the old man to the limit of the table, not that we had any doubt about his heirs getting the money, but we desired to see how much cash he would have to pay, and how much of his money would go direct into the hands or pockets of the share-holders. It appears that he paid in cash $9,793.40, of which the share-holders, under their charter rights, took $1,652.00 into their own private possession. The policy-holder in the All-cash Company, as we have seen before, had fully paid up at the end of the thirty-eighth year, and had paid in all but $4,077.94, after which his policy increased progressively in amount, or, at his option, it yielded him a yearly income. We find in the whole transaction the share-holders in the Cash Company, although handsomely paid for their attention to the business, received only $73.76, and this was from interest on the additional guarantee fund, which belongs to the policy-holders in that Company—is held for the security and benefit of the policy-holders, and yields sufficient interest to pay the share-holders a VERY LIBERAL, but not exorbitant, compensation for their personal attention to the business of the Company.

COMMENTS UPON INSURANCE PARTLY ON CREDIT.

A Life Insurance Company can, with safety to itself, accept the notes of a policy-holder in part payment of the "net annual premium," and the amount of these "notes" or "loans" may equal, but must not exceed, the Deposit. The Deposit increases from year to year, and the notes or loans may be increased to the same extent, but no more. The notes or loans must be deducted from the face of the policy at maturity, therefore, *the amount actually insured in money* becomes less and less each year. The question is not, "CAN A LIFE INSURANCE COMPANY SAFELY ACCEPT NOTES IN PART PAYMENT OF THE ANNUAL PREMIUMS?" But rather, "CAN A POLICY-HOLDER, FOR ANY GREAT LENGTH OF TIME, AFFORD TO ACCEPT THE CREDIT PROFFERED BY THE LIFE INSURANCE COMPANY?"

Suppose that we take this case to the limit of the table, ninety-nine years. The policy-holder will have paid each year, his proportion of the losses by death—called Cost of Insurance—and the yearly expenses; and the Deposit, consisting entirely of his own NOTES, will have amounted to within a very small fraction of the whole amount of the face of his policy. The man dies in the one hundredth year of his age, and the heirs receive his NOTES in part payment of the policy; and THESE NOTES ARE, IN THIS PARTICULAR CASE, ENOUGH TO FULLY PAY THE POLICY WHEN THE LAST ANNUAL PAYMENT ONLY, IN MONEY, IS ADDED TO THESE NOTES.

This certainly is not a desirable kind of Life Insurance for those who live long. On the other hand, if the insured dies early, he will gain by the note, or loan system; but what he gains by notes or loans, the man in that Company who lives long loses.

The Life Insurance Company is safe in this case, *provided* it has a large number of policy-holders, and *retains* them to the end of their lives.

It is true that NOTE or LOAN Companies seldom, if ever, in practice, push the credit system to the extreme limit given above; but they may do it with safety under the above *proviso*. The question is, can the policy-holder stand it if he does not die soon?

RETURN PREMIUM PLAN.

There is a comparatively new, and, at first glance, to some persons, attractive kind of whole Life Insurance, by which the Company contracts not only to pay the amount of the policy at maturity, but also to return, at the death of the policy-holder, the full amount of all the premiums he has paid.

This, of course, cannot be done by the Company unless the policy-holder PAYS for all he gets. It appears by the calculations made in these cases for the return of premiums, that it requires $53.65, net annual premium, to insure $1,000 at age forty-two and return the premium. But $25.55 is the net annual premium that will insure $1,000 at the same age on the ordinary life policy. Twice this amount, or $51.10, will insure $2,000 for the first year, whereas $53.65, on the Return Premium Plan, will only insure for the first year $1,053.65. It is hardly necessary to continue the discussion of the Return Premium Plan through its mathematical complexities. It makes too bad a commencement at Life Insurance, and a man would have to be certain of living nearly a quarter of a century before he could hope to gain any thing by this plan.

GENERAL COMMENTS.

It is essential to the policy-holder that the Life Insurance Company with which he may take out a policy, should be controlled by wise and stringent laws, rigidly enforced; because, from the nature of this business, the funds held in trust are peculiarly liable to misapplication. Nothing short of the searching probe of stringent and wise laws, rigidly enforced, can prevent this, in case the managers of such Companies are not thoroughly acquainted with the principles upon which Life Insurance is founded, and firmly determined to adhere to them. They must attend closely to every detail; make no mistakes in their risks or investments; and should be men of the greatest integrity of character and sternest honesty of purpose. But, to insure safety in the business, every detail should be furnished, at least once in every year, to some competent State officer; and by the latter, the accounts should all be carefully recomputed, and the results published. Sound and well-conducted Companies desire this, and others should be forced to a full exhibit of all their affairs.

Borrowing Great Names.

"The device of borrowing great names has, from the commencement, been resorted to by the projectors and managers of many Life Insurance Companies. The facility and readiness with which men of influence lend their names to business, of which they know little and care less, has long been, and is yet, a short and easy road to temporary popularity and public favor for business, that has no sound and legitimate claim to the confidence and respect of the community." An array of great names in Life Insurance is not enough. Neither is it a very sound business principle to trust, in a matter of this importance, to the mere request or solicitation of an agent. It is not enough that the agent may point out in a table of figures the amount a person would be required to pay for a certain kind of policy, and in his own mind figure up what his (the agent's) commission would amount to IN CASH, on the premium he asks the policy-holder to pay to him for the Company. There is more sense than this would indicate in the practical business of Life Insurance.

NUMERICAL BRAGGING.

In addition to what has been said of the use made of great names in connection with this business, it is stated by writers on Life Insurance, that, "for time out of mind, the practice of NUMERICAL BRAGGING has been by some Companies carried to a high pitch of extravagance; and such Companies rely for public favor rather upon the authority of great names than upon a full, frank, and conclusive exhibition of their affairs."

It appears that, although safe investments cannot be made in this country at rates of interest higher than from seven to eight per cent. per annum, some Life Insurance Companies promise, and even guarantee, to pay to policy-holders thirty, forty, and as high as fifty per cent. DIVIDENDS upon the premiums. Bear in mind that the Company has to pay all the current expenses of the business; to pay the losses by death during the year, and set apart and retain for investment the requisite Deposit for the policy, before it can make any dividends. As the Company can only safely make seven or eight per cent. interest upon the funds intrusted to it, these *enormous dividends* to policy-holders, after meeting the above obligations, look like numerical bragging.

Let us see a little further into this enormous dividend. In the first place, the expenses of Life Insurance Companies are large. Agents' commissions, salaries of officers, traveling expenses, taxes, printing, rents, stationery—these, and other expenses, have to be paid IN CASH. The losses that occur during the year, by death, must be paid IN CASH. The expenses and the losses by death are paid by the Company; BUT THIS IS DONE WITH THE MONEY OF THE POLICY-HOLDERS.

The Deposit is a specific amount, determined by accurate arithmetical calculation; this amount must be in the hands of the Company, and held securely invested at a certain rate of interest, and this interest regularly compounded every year, in order to enable the Company to pay its policies at maturity in money. If the Company has in its hands the requisite Deposit, it is solvent. If it has not on hand, and securely invested at the fixed table rate of interest, the full amount requisite for the Deposit, the Company cannot pay its policies in money at maturity. Of course the Company must retain the requisite Deposit for each and every one of its outstanding policies; must pay current expenses; must pay the losses that occur by death, each year, of a certain number of policy-holders; and as the Company can only make seven or eight per cent. by safe investments of the funds intrusted to it by the policy-holders, the ENORMOUS DIVIDENDS so much talked of look like NUMERICAL BRAGGING.

THE DEPOSIT IS NOT CASH CAPITAL.

When a purely Mutual Company advertises $12,000,000 CASH CAPITAL, and an examination of the official reports show that nearly $10,000,000 of this Cash Capital is the Deposit—an accrued liability—a debt; and that the Company would be insolvent if it had not the means on hand to pay this debt, nearly $1,000,000 of the Cash Capital proves to be premium notes paid to the Company by policy-holders in lieu of money; and something more than $1,000,000 turns out to be Surplus, belonging to, but withheld from, individual policy-holders. It would seem that the Numerical Bragging alluded to by former writers, is still practiced by some Life Insurance Companies, when they convert $10,000,000 of debt, $1,000,000 in premium notes, and $1,000,000 of retained Surplus, into $12,000,000 CASH CAPITAL.

ASSETS THREE TIMES THE LIABILITIES.

Some Companies boast of assets amounting to three times their liabilities. It should be borne in mind, that when the insured pays his annual premium, the Company at once becomes liable for the expenses for the following year for the Cost of Insurance or losses by death during the year, and for the Deposit at the end of the year. Assets amounting to three times the liabilities of a Company, indicate a bad case of *numerical bragging;* and even the authority of great names cannot long give weight and influence to Companies that present statements of the character referred to above, viz: Enormous Dividends—Purely Mutual Company—with $12,000,000 Cash Capital, and assets of a Life Insurance Company amounting to three times its liabilities.

No Dividends from Net Premiums at Net Interest.

What is called the Net Premium in Life Insurance is a matter of direct arithmetical calculation, based upon a Mortality Table which gives the law of duration of human life when applied to large numbers of mankind. The Mortality Table is based upon statistical facts. In these calculations a rate of interest is taken so low that it may safely be assumed that this interest will be realized and regularly compounded every year during the existence of the contract between the policy-holder and the Life Insurance Company. This period is generally for the lifetime of the policy-holder.

The net annual premium for each policy, as above determined, is enough, and only enough, to pay the losses that occur each year by the death of policy-holders, and provide the requisite Deposit, at the end of each year, which will enable the Company to pay all its obligations at maturity. No dividends need be expected then from the net premium at net or table interest.

Policy-holders Should Investigate Certain Points.

Beyond what the law can do for a policy-holder, it is well that he attend, either himself or through a competent person on whom he can rely, to several matters bearing on the question of Life Insurance. "Great names" and high business qualifications in other professions, are not in themselves sufficient to conduct Life Insurance successfully. The question is, do the officers of the Company comprehend the principles upon which the business is founded; and do they give their own close personal attention to it. If they do not understand and closely attend to the business, their high character, business capacity, and great names, are a delusion in Life Insurance. If the officers are, in every respect, the right men for this most serious, important, and gigantic business, it is well to look further and inquire closely into the terms and conditions of the contract between the Company and the policy-holder. These are expressed on the face of the policy; and in some Companies are liberal and just; in others they are vexatiously and harshly restrictive, not to say unjust. It is but a few years since it was the universal practice of Life Insurance Companies to appropriate to themselves the whole accrued value of a policy in case the holder thereof failed on a given day to pay his annual premium.

Suppose that the old man, whose account we followed, year by year, from age thirty, to include age ninety-nine, had failed to pay his last annual premium: under the rule followed by all Life Insurance Companies, only a few years ago, his Deposit, amounting to $9,445.70 on a policy of $10,000, would have been declared forfeited, and the Company would have pocketed this money. Bear in mind that this policy had contributed its proportion to pay the losses by death of other policy-holders every year; had paid its proportion of the yearly expenses, and that $1,652 of this old man's money, not including interest, had already gone direct into the private possession of the share-holders, without in the slightest degree touching the matter of his insurance. Only a few years ago this Deposit, produced entirely by the policy-holders' money, would have been confiscated by the Company, thus robbing the weak and unfortunate to increase the *dividends* of the strong and rich. There was no justification or excuse for this rule of forfeiture for non-payment of premiums except in the fact that this was a condition expressed in the contract. That it continued for so long a time to be the universal custom can only be accounted for by the fact that the principles upon which Life Insurance is founded were not thoroughly understood by business men. The terms of the contract require attention, because the quality of the article purchased by the policy-holder, *i. e.* the policy, is directly affected by these terms, as well as by the considerations previously mentioned. Now we come to the question of price charged. There can be no safety or certainty of the payment of policies at maturity, and, therefore, no real insurance, in case a Company charges less than the net annual premium, and enough more to cover the expenses. Because, in spite of all we hear about large "*dividends*" to policy-holders, arising from the "*investment*" of premiums, the net annual premium and net interest upon it must go to effect the insurance, and the expenses must be paid in addition. It appears from this view of the case, that a Life Insurance Company may charge too little. It may do this and be irretrievably insolvent, and still give no external sign of its condition for the lifetime of a generation, because, in the first thirty or forty years of the existence of a Company, the annual premiums are largely in excess of the death claims.

CERTAINTY OF THE PAYMENT OF HIS POLICY AT MATURITY IS WHAT EVERY POLICY-HOLDER WANTS. To insure this, it is necessary that the Company should charge enough to enable it to meet all its liabilities during the worst year that may reasonably be expected to occur during the continuance of the contract; and this is generally for a lifetime. Therefore, when the mortality is greatest, and the interest on investments lowest, and expenses heaviest, the Company must have the means of meeting its liabilities. It follows that, in favorable years, there will be an over-payment. In case this over-payment is all returned to the policy-holder at the end of the business of the year, it would seem that it is not a matter of vital importance whether the premium is a little more or a little less, provided it is enough to make the payment of the policy at maturity certain; and provided further, that the business of the Company is managed with ability, integrity, and economy; that the trust is well administered in every respect, and the policy is liberal and just in its conditions. But if a limited number of share-holders, with capital stock, say of $100,000 or $200,000, are by their charter authorized to appropriate to themselves twenty, or even five, per cent. of the surplus arising from the over-payments made by a large number of policy-holders, these capitalists will make enormous profits out of the funds a man sets apart, whilst living, for the protection of his widow and orphans from poverty and want that might befall them, in case of his early death. On a really *flourishing business*, two per cent. of the over-payments made by a large number of policy-holders, will make "*enormous profits*" FOR THE SHARE-HOLDERS.

In 1868 an Insurance Company returned to its policy-holders $3,000,000 of surplus arising from over-payments made during the year by the policy-holders. This was a purely Mutual Company. Suppose that a Company, with $100,000 of capital stock, succeeds in attaining a like flourishing business, and that the share-holders of this Company have secured to themselves, by special charter, the perpetual right to appropriate to themselves twenty per cent. of all the surplus arising from over-payments made by the policy-holders, the "Happy Capitalists" of such a Company would receive $600,000 per year in addition to legal interest upon their $100,000 of full-paid stock. It is hardly necessary to tell the policy-holder that it is his money that produces these "enormous dividends" for the share-holders. The "Sacred Fund" set apart by our "friend and neighbor" during his life, for the purpose of protecting his family from poverty and want after his death, is hardly the source from which capital should seek to make six hundred per cent. per annum. Capital, however, is proverbially aggressive, and will not hesitate to take twenty per cent. of the surplus of the policy-holders, *provided the latter agree to it.* The per cent. of surplus secured to share-holders by their charter is a matter upon which no prudent policy-holder should fail to inform himself. On the other hand, it is not always safe to assume that the purely mutual and entirely philanthropic basis is necessarily the best reliance in a business sense. Even the purely Mutual Companies do not, as a rule, distribute all their surplus; nor do the policy-holders in such Companies get this business attended to for them for nothing.

The share-holders of some of the mixed Companies receive no other compensation for the use and risk of their capital, and their own personal attention to the business, than the legal interest earned by that capital. This is rather too generous on their part; but it may be that the share-holders find sufficient compensation in the honor and consequence arising from their being the custodians, and having the permanent control and handling, of millions of dollars in trust for other people.

It is believed by many that the purely Mutual Companies are defective in an essential particular. They offer no adequate inducements for the best business men to become trustees, and to devote the same attention and energy to the business of the Company that they would give to their own personal and private affairs. It is true that many of the purely Mutual Companies have competent officers, and give them handsome compensation for their attention to the business of the Company; but after all, this is a salaried service, which, by business men is not, as a general rule, considered as safe a reliance as that of personal ownership and money of share-holders staked upon the success of an enterprise.

It is stated by some writers that the plan of compensating share-holders, by allowing them legal interest upon a limited amount of an additional guarantee fund, formed gradually by retaining in the hands of the Company a portion of the Surplus, "*identified the interests of the stock-holders and policy-holders, and thus guarded both, while it tended to reduce largely the average of the Company's losses and expenses.*"

Insurance Partly on Credit.

Many persons insist that it is cheaper, safer, and better for men to insure their lives partly on credit, than it is to insure on the all-cash plan. This note or loan system of Life Insurance has strong advocates amongst well-informed insurance writers. But in the long run policy-holders will find there is some delusion about the credit so generously proffered and urged upon their acceptance. It is true that if a man is certain that he will die soon, and he can get $100 worth of insurance for $50 in cash and his note for $50, he would do well to take out a policy in a note Company, die during the year, and let his heirs receive the amount of the policy, less his note for $50; but there are many and strong reasons why the system of Note or Loan Life Insurance is not advantageous to those who continue to renew their policies in such Companies for any great length of time.

Proprietary or Purely Stock Companies.

Besides the purely Mutual Companies and the mixed Companies, there are Companies conducted on the strictly proprietary plan. These *purely Stock* Companies, in which all the surplus belongs to the share-holders, as a general rule, charge less premiums than the purely Mutual or mixed Companies; but they return no surplus to policy-holders—their theory is, that THEY MAKE DIVIDENDS TO THEIR POLICY-HOLDERS IN ADVANCE, by charging less premiums. The fact is, that *dividends* to holders of Life Insurance policies are simply a return of that part of the annual premium which was paid to the Company at the beginning of the year, and which, at the end of the business of the year, is found not to have been required in paying the expenses, paying the losses by death, and providing the requisite Deposit at the end of the year. THE REAL DIVIDENDS IN LIFE INSURANCE—THAT IS TO SAY, REAL INCOME PRODUCED BY THE INVESTMENT OF CAPITAL IN BUSINESS—ARE MADE TO SHARE-HOLDERS, *and these, as seen above*, ARE SOMETIMES ENORMOUS.

A great deal depends upon the capacity, good judgment, close personal attention and integrity of the officers in control; but it may be assumed as certain that a correct knowledge of the principles upon which the business is founded—that is to say, the peculiar arithmetical law, applicable to these future and contingent values—should be clearly understood; and it is hardly possible for any man to conduct this business successfully without understanding clearly the principles upon which the calculations of values and the liabilities incurred are based.

Great names alone will not answer the purpose; nor will numerical bragging command ultimate success.

Accumulation Necessary in the Earlier Years.

One of the most striking features in the practical business of Life Insurance, as at present generally conducted, arises from the fact, that, in the early years of a policy, it is necessary to accumulate money, in order to meet the demands arising in the later years, when the death claims will so largely exceed the premiums. This causes these Companies to be the custodians of millions of money; and all these millions must be regularly and safely invested.

May be Made Safe, but it Needs Watching.

Life Insurance may, from its peculiar nature, be made, perhaps, the safest business known—at the same time, in the hands of those ignorant of its principles, or incompetent to control, even if they understand it, it can go further wrong, and show less evidence for years of its utter insolvency, than any other business ever devised. And whilst it may be made the safest, it is a business in which, if it is not thoroughly comprehended and strictly guarded, designing fraud may raise a curtain behind which its worst schemes can be carried on free from detection, until such time as the death claims exceed the annual premiums; that is to say, for thirty or forty years.

To fully appreciate this fact, it is only necessary to recall the illustration previously given, in which it was seen, that, at the end of the thirty-fourth year, nearly $28,000,000 was on hand in Deposit, after paying all the death claims that had previously matured. This sum, and all the future net annual premiums, with compound interest on the whole, is required in order to enable the Company to meet its liabilities. Suppose that this $28,000,000 had been appropriated to other purposes? This might have been done, and the Company have paid all its losses up to that time, and, to external appearance, have seemed all right; and this, too, with a real defalcation of $28,000,000.

One Hundred Thousand Dollars Deposited is of but Little Avail in Certain Cases.

It is no doubt, upon good grounds, that the law of some States requires that $100,000 of capital stock should be paid up and deposited with the Treasurer of the State before a charter is granted to a Life Insurance Company.

But it is well that policy-holders and all others interested should know the fact that, in a Company with a large number of policies of long standing, capital stock of $100,000, or even $1,000,000, in the hands of a State Treasurer, would not secure the payment of premiums in that Company, in case the DEPOSIT HAD BEEN MISAPPLIED, LOST, OR STOLEN. Capital stock in a Life Insurance Company that has succeeded in securing a moderate number of policy-holders, is of but little avail except in securing for the Company the close personal attention of business men *to their own private interests.*

But to effect this, it is not considered that it is absolutely necessary to give these share-holders an easy opportunity for making six hundred per cent. per annum upon their stock.

CUSTODIANS OF IMMENSE SUMS OF MONEY.

Life Insurance Companies, by their intrinsic nature, must become, if moderately successful in securing business, the custodians and investors of immense sums of money. The location of a Company, as well as the experience, capacity, integrity and industry of those who manage the business, is a subject for close consideration. But it should be borne in mind, that, because ten per cent. is legal interest in one State, and six per cent. is the legal interest in another, it does not *necessarily* follow that a Company chartered in the latter may not invest its funds in the best safe market just as well as the former.

AGENTS' COMMISSIONS.

The large per cent. of the premiums paid to agents is an item of very heavy expense to Life Insurance Companies. And another great expense is the publishing of a large amount of what is called "Campaign Literature." It is perhaps impracticable for the Companies to materially lessen these enormous expenses, so long as the present extraordinary competition is kept up, and THE PUBLIC ARE UNINFORMED IN REGARD TO THE TRUE PRINCIPLES UPON WHICH THE BUSINESS OUGHT TO BE CONDUCTED.

If policy-holders had clear and distinct ideas of their own in regard to Life Insurance, and would seek for the best article at the fairest price, in this business, as they already do in regard to their other purchases, the best Companies would no doubt be but too glad to abate from their premiums that portion of the "Loading" which now goes to pay these large commissions to agents. And IT WOULD NO LONGER BE PROFITABLE FOR THE AGENTS TO REPRESENT THAT, BY BUYING AND PAYING FOR A LIFE INSURANCE POLICY, THE POLICY-HOLDER IS, IN ADDITION, MAKING AN INVESTMENT THAT WILL PAY HIM FORTY PER CENT. "DIVIDENDS;" and this, too, after the Company loans the policy-holder one third or one half the premium, and charges him but six per cent. for the accommodation.

A Large Deposit or Reserve Means a Large Debt.

The large Companies that have outstanding policies, upon which the Deposit has been accumulating for years, are supposed by the general public to be IMMENSELY RICH. And many, believing that this is really true, find in it sufficient reason for the pertinacity and importunity with which they are urged by certain parties to insure their lives. The Companies that have large Deposits are not *rich;* but they are the custodians of immense sums of money, belonging, it is true, to others, but invested and handled by the Company. This is in itself a matter of immense moment. Add to it the enormous cash dividends to share-holders that must arise in Companies that have acquired by charter the right to appropriate twenty per cent. of the surplus arising from over-payments made by the policy-holders, and it is not difficult to see and understand why men are pressed to patronize their "friends and neighbors." If the Company can once attain a "flourishing business," the "Happy Capitalist" will get six hundred or seven hundred per cent. yearly dividends on his stock in addition to the power and other incidental advantages to the Company, arising from the fact that it handles and controls millions upon millions of money.

In the current business of each year, large amounts are received from premiums, large amounts are paid out for losses by the death of policy-holders, and a vast deal goes to pay the ordinary expenses; but in old-established companies, that have large numbers of policies that have been in force for years, the TRUST FUND DEPOSIT is the great item. And it is upon the security and safety of this *fund* that the ultimate ability of a Life Insurance Company to pay all its policies at maturity mainly depends.

Not Exempt from the Usual Results of Mismanagement in Ordinary Business.

Notwithstanding the accuracy of the theory upon which the business of Life Insurance is founded, there are many contingencies that may prove fatal to Companies in practice; and whilst strict compliance with certain fixed principles and definite rules will always enable a Company to pay its policies at maturity, there are many things that will, if permitted to occur, bankrupt a Life Insurance Company just as certain as a disregard of the peculiar laws governing this business will lead the Company ultimately to inevitable destruction. These Companies are not exempt from the effects produced by dishonesty, fraud, and defalcation. Moreover, continued lavish expenditures, the selection of bad risks by insuring impaired or unhealthy lives, or making unsafe investments, will lead to bankruptcy certain.

There can scarcely be any saying more groundless than the statement often heard, that "Life Insurance Companies cannot break." And, on the other hand, it is absurd to say, that, when well-conducted in every particular, it is impossible for Life Insurance Companies to comply with all their obligations, and pay all their policies at maturity. The plain fact of the case is, that LIFE INSURANCE COMPANIES CAN BREAK, AND WILL BREAK, UNLESS MANAGED WITH SKILL AND INTEGRITY. On the other hand, it is undoubtedly true that THE BUSINESS OF LIFE INSURANCE CAN BE MADE MORE SAFE AND MORE SECURE THAN ANY OTHER COMMERCIAL BUSINESS KNOWN AMONGST MEN. But honest ignorance *cannot*, and designing fraud *will not*, effect this result. Capacity, integrity, industry, skill, and sound judgment on the part of those in control, are just as essential to success in Life Insurance as they are in all other kinds of important business.

The Principle Upon which Money Values in Life Insurance are Calculated Should be Understood.

The mere fact that a man can compute interest on money will not make him a competent banker, neither will a knowledge of the formulas and rules be in itself sufficient to fit a man for the important business of Life Insurance. But it would be far better to intrust banking to men who cannot calculate interest on money, than to intrust Life Insurance to those who are not acquainted with the method upon which calculations of important money-values in this business are based.

There is danger to all in the doctrine *often promulgated by agents*, that Life Insurance business can be better conducted by men who do not understand the "art of calculating these values" than by those who do understand the principles upon which alone this business can be safely conducted. Those who talk in this way are, generally speaking, FORTY PER CENT. DIVIDEND MEN, who propose to lend one third or one half the premium to the policy-holder at six per cent.; and promise him forty per cent. DIVIDEND per annum upon the whole amount of the premium. The same persons generally style the money of the policy-holder that is held by the Company in trust for the purpose of enabling it to pay the policy at maturity, CASH CAPITAL; or, at least, announce millions of ASSETS, and are silent about these assets being a deposit debt, held by the Company in trust for other people.

Companies not so Rich as Some People Suppose.

It is often urged by intelligent business men who are not acquainted with the real nature of the Trust Fund Deposit (or "Reserve"), that the Life Insurance Companies have made vast sums of money, and that the policy-holders must have furnished the money that made these Companies so immensely rich. This view of the subject has been already answered by the statement that this Deposit does not belong to the Company, but is, in fact, the money of the policy-holder, held by the Company in trust.

Life Insurance Companies Great Money Lenders.

It is often urged, too, that Life Insurance Companies are absorbing a very large portion of the currency of the country; and many persons seem to apprehend that this will result in extraordinary scarcity of money. But it must be remembered that Life Insurance Companies are compelled to keep their funds constantly invested; they are, therefore, forced to be lenders of money; and, as a general rule, they are more careful about the character of their securities than anxious to realize exorbitant rates of interest.

In Practice, Payment not Postponed to the End of the Year.

Although in theory the amount of a policy is not due until the end of the policy year within which the insured may die, it is usual for Life Insurance Companies, in practice, to pay the policy within from thirty to ninety days after proof of the death of the policy-holder.

Policy-Holder Assumed to be Aged an Exact Number of Whole Years.

It is usual to assume that a person who applies for insurance is exactly a given number of years old. The Mortality Tables and the calculations are based upon whole years; and the age is taken to be the whole number of years nearest to the real age. For instance, if the real age of a person was thirty years and five months, he would be considered thirty years old; but if the real age was thirty years and seven months, he would be taken as thirty-one years old.

Extreme Haste in Advertising the Payment of One Policy.

There is a great deal of what, in common parlance, is called "clap-trap," resorted to by some Life Insurance Companies and agents. Besides what has been previously stated in reference to the borrowing of great names, the enormous dividends to policy-holders, and numerical bragging in general, attention is here called to the extreme *haste* with which some Companies and agents rush the announcement into the newspapers, *hurriedly advertising the fact that they* HAVE PAID A POLICY *at maturity;* and seem to offer an isolated fact of this nature as proof *positive* that the Company will pay all claims that may hereafter mature against it. What would be thought of a bank that took especial pains to herald in the newspapers that it had paid *one* of its obligations, and called upon the community to accept this as proof that it was solvent and would stand so forever?

Difference Between the General Law upon which the Risk is Based in Life Insurance, and the Nature of the Risk in Fire Insurance.

As previously stated, more than once, in these Notes, the general law regulating the duration of human life has been very accurately determined. Upon this law, and an assumed safe rate of interest, all Life Insurance calculations are based. Notice the contrast between this and the data upon which the calculations or estimates in the business of Fire Insurance are founded. It was stated by the National Board of Fire Underwriters, in a report dated 1868, that, "*as a whole, the business* (of Fire Insurance) *is absolutely without that chart of experience, furnished only by combined results, carefully noted and preserved.*" (See New York Insurance Report, 1868, page 7.) Notwithstanding this, nearly every prudent business man insures his property against destruction by fire; and this, too, when there is a strong probability that his property will never be destroyed by this cause. It is certain that all men must die. The business of Life Insurance *can be made safe;* and yet there are very many men who have not insured their lives, and have no present intention to do so. In many cases this probably arises from a belief that the system itself does not rest on principles and laws that are certain and stable; and in other cases it perhaps arises from some apprehension that the system may not be fairly and honestly administered.

Further Allusion to the General Law of Duration of Human Life—Medical Examiners.

Ordinary business, in times of panic or extra stringency in the money market, is liable to a sudden strain upon its resources that often proves fatal; whereas, in Life Insurance, particularly in those Companies that have large numbers of policy-holders judiciously selected and distributed, experience has proved that the general law of duration of human life holds true with remarkable regularity; and for this reason Life Insurance Companies are to a great degree exempt from those sudden and extreme demands upon their resources, which are at many periods so fatal to ordinary commercial business.

But the general law governing the duration of human life will be of little or no avail in case a Life Insurance Company accepts risks upon impaired or diseased lives; and Companies that have only a small number of policy-holders will always be, to some extent, liable to a number of losses not in accordance with the general law of duration of human life; because this law only applies to large numbers, not to a single individual, or to a small number of individuals. Much of the success of Life Insurance Companies depends upon the skill and integrity of the Medical Examiners.

Lawyers—Professors of High Schools—Accountants.

Lawyers should understand the true principles of Life Insurance, because $2,000,000,000 was never yet staked upon any one business without the subject being sooner or later brought into the courts.

Professors of High Schools should understand the principles of Life Insurance, because this business has already attained magnitude, such that every intelligent, educated man in the country ought to have a correct knowledge of its nature and bearing upon the general welfare.

Educated accountants, book-keepers, clerks, and computers, ought to understand the manner in which these accounts are made, and the values calculated. They may all rest assured that there is nothing in the theory, the principles, or in the method of calculating values in Life Insurance, that is not within the easy comprehension of men who have a thorough knowledge of the single rule of three, and a *mere acquaintance* with the simplest principles of elementary algebra. The intelligent, general business sense of the country should be informed definitely what Life Insurance is, and what it is not; and it is hoped that the foregoing *Notes* may tend, in some degree, to promote this end.

"In a body of lives of the same age, all selected as healthy from the general mass of mankind, it is obvious that the rate of mortality must be considerably less *for the first ten or twenty years after selection, than amongst those from whom they are thus chosen;* as, however, these selected lives advance in age, their general health, and the rate of mortality amongst them, will naturally approximate to the common standard."—MORGAN.

"One is struck with the fact that assured lives are, for some time after selection, much better than the community at large, *but that after awhile they become much worse.* This can arise from no other cause *than the selection which the assured exercise against the Company* by dropping policies on healthy lives, and retaining those on lives which have become bad or doubtful."—HIGHAM.

"Those persons will be most for flying to these establishments who have feeble constitutions, or are subject to distempers which they know *render their lives particularly precarious; and it is feared that no caution will be sufficient to prevent all danger from hence.*"—DR. PRICE.

"The necessity of the valuation to an effective supervision, arises from the peculiar nature of the business of Life Insurance. In this peculiarity lies its greatest danger. The opportunity for fraud or fatal error. Life Insurance reverses the laws which govern all other commercial enterprises and investments. In the latter the expenditure comes first, and the profits, if any, come afterwards. In the first years of a Life Insurance Company, its treasury overflows with the incoming premiums, whilst its liabilities are postponed for the lifetime of a generation. For more than thirty years it furnishes a constant margin for plunder or perversion of its funds, while its ultimate failure, though certain if the opportunity is improved, is still remote. Unless its condition is probed by some decisive test, it exhibits no necessary symptoms of its insolvency until the claims by death begin to equal or exceed the premium receipts; and this period will not ordinarily be reached until nearly forty years from its start. If it can once be fairly believed that there is no mystery surrounding the process technically called valuation, too deep for ordinary ken, its reasons and importance may be better, or at least more generally, understood."

WILLIAM BARNES.

GENERAL SUMMARY OF FORMULA AND ARITHMETICAL RULES FOR LIFE INSURANCE NET CALCULATIONS.

NET SINGLE PREMIUM TO INSURE ONE DOLLAR FOR WHOLE LIFE.

$$sP_x = 1-(1-v)A_x.$$

RULE.—*Subtract v from 1; then multiply by A_x, as found in the tables,* $\left(\text{or, which is the same thing, multiply by } \frac{N_x}{D_x}\right)$*; and subtract the product from one dollar; the remainder is the net single premium that will insure one dollar for whole life at the age x.*

NOTE.—v, in all cases, is equal to 100, divided by 100, plus the table or net rate of interest,

NET ANNUAL PREMIUM TO INSURE ONE DOLLAR FOR WHOLE LIFE.

$$aP_x = \frac{1}{A_x} - (1-v).$$

RULE.—*Subtract v from 1; subtract this remainder from the quotient obtained by dividing 1 by A_x. The result is the net annual premium that will, at age x, insure one dollar for whole life.*

TO DETERMINE THE AMOUNT OF THE TRUST FUND DEPOSIT (OR RESERVE) AT THE END OF n YEARS ON A WHOLE LIFE POLICY FOR ONE DOLLAR, TAKEN OUT AT THE AGE x.

$$(\text{T. F. D.})_{x+n} = 1 - \frac{A_{x+n}}{A_x}.$$

RULE.—*First find from the table the value of large A at the age $x+n$; then the value of large A at the age x; divide the former by the latter, and subtract the result from one dollar.*

GENERAL FORMULA FOR CALCULATING THE TRUST FUND DEPOSIT (OR RESERVE) AT THE END OF n YEARS ON ANY KIND OF POLICY FOR ONE DOLLAR, TAKEN OUT AT AGE x.

$$(\text{T. F. D.})_{x+n} = u_{x+n-1}\left((\text{T. F. D.})_{x+n-1} + aP_x - c_{x+n-1}.\right)$$

RULE.—*Calculate aP_x by the formula applicable to the case; add to this the Deposit (or Reserve) for the year n—1; then subtract from this sum c_{x+n-1} (obtained from the table); and multiply the remainder by u_{x+n-1} (obtained from the table); the result is the Deposit (or Reserve) for a policy of one dollar at the end of n years from its date.*

TERM INSURANCE.

NET SINGLE PREMIUM THAT WILL INSURE ONE DOLLAR FOR n YEARS.

$$sP_{x|n}=\frac{v(N_x-N_{x+n})-(N_{x+1}-N_{x+n+1})}{D_x}.$$

RULE.—*From the tables find the value of* N *at the age x; then find the value of* N *at the age $x+n$; subtract the latter from the former, and multiply the remainder by v. Then find from the tables the value of* N *at the age $x+1$, and subtract from this the value of* N *at the age $x+n+1$. Now subtract this remainder from the amount obtained by multiplyiug v by (N_x-N_{x+n}); and divide this remainder by* D *at the age x (obtained from the table); the result is the net single premium sought.*

NET ANNUAL PREMIUM THAT WILL INSURE ONE DOLLAR FOR n YEARS.

$$aP_{x|n}=v-\frac{N_{x+1}-N_{x+n+1}}{N_x-N_{x+n}}.$$

RULE.— *Find from the table the value of* N *at the age $x+1$; subtract from this the value of* N *at the age $x+n+1$. Then divide this remainder by the remainder obtained by subtracting* N *at the age $x+n$ from* N *at the age x. Subtract the result of this division from v, and we have the net annual premium that will, at age x, insure one dollar for n years.*

NET SINGLE PREMIUM FOR AN ENDOWMENT OF ONE DOLLAR AT THE END OF n YEARS.

NOTE.—*This is endowment pure and simple, and does not include insurance.*

$$E_{x|n}=\frac{D_{x+n}}{D_x}.$$

RULE.—*Find from the table the value of* D *at the age $x+n$; and divide this by the value of* D *at the age x.*

NET ANNUAL PREMIUM FOR AN ENDOWMENT OF ONE DOLLAR AT THE END OF n YEARS.

$$\frac{E_{x|n}}{A_{x|n}}=\frac{D_{x+n}}{N_x-N_{x+n}}.$$

RULE.—*Find from the table the value of* D *at the age* $x+n$; *and divide this by the* DIFFERENCE *between* N *at the age* x *and* N *at the age* $x+n$.

NET SINGLE PREMIUM FOR INSURANCE OF ONE DOLLAR FOR n YEARS, AND AN ENDOWMENT OF ONE DOLLAR AT THE END OF n YEARS.

$$(sP+E)_{x|n} = \frac{N_{x+n}+v(N_x-N_{x+n})-N_{x+1}}{D_x}.$$

RULE.—*Find from the table the value of* N *at the age* x; *subtract from this the value of* N *at the age* $x+n$; *multiply the remainder by* v, *and add this product to the value of* N *at the age* $x+n$; *then subtract from this sum the value of* N *at the age* $x+1$; *divide the remainder by* D *at the age* x, AND WE HAVE THE NET SINGLE PREMIUM THAT WILL, AT AGE x, INSURE ONE DOLLAR FOR n YEARS, AND AT THE SAME TIME PROVIDE FOR AN ENDOWMENT OF ONE DOLLAR TO THE INSURED AT THE END OF n YEARS, IN CASE HE IS ALIVE AT THAT TIME.

NET ANNUAL PREMIUM FOR INSURANCE AND ENDOWMENT OF ONE DOLLAR AS ABOVE.

$$\frac{(sP+E)_{x|n}}{A_{x|n}} = v - \frac{N_{x+1}-N_{x+n}}{N_x-N_{x+n}}.$$

RULE.—*Find from the table the value of* N *at the age* $x+1$; *subtract from this the value of* N *at the age* $x+n$. *Then divide this remainder by the difference between* N *at the age* x, *and* N *at the age* $x+n$. *Subtract this result from* v, AND WE HAVE THE NET ANNUAL PREMIUM IN THIS CASE.

THE NET ANNUAL PREMIUM FOR n YEARS THAT WILL INSURE ONE DOLLAR FOR WHOLE LIFE IS EXPRESSED BY—

$$\frac{D_x-(1-v)\,N_x}{N_x-N_{x+n}}.$$

RULE.—*Find from the tables the value of* N *at the age* x; *multiply this by* $(1-v)$; *subtract the product from* D *at the age* x; *and divide the remainder by the difference between* N *at the age* x, *and* N *at the age* $x+n$. This rule gives the net annual premium that will, if paid annually for n years (provided the insured is alive to make the payment), insure one dollar to be paid to the heirs of the insured at the end of any year in which he may die.

Explanation of the Manner in which the Tables Inserted in this Work were Constructed.

Table I. The Mortality Table adopted in the foregoing "Notes," is the result of experience in seventeen leading English Companies; and is generally called "The Actuaries'."

The net interest is taken at four per cent. per annum. v, in this case, is equal to 100, divided by 104; or $0.961538.

The quantity we have represented by A_x is equal to a fraction, the numerator of which is represented by N_x, and the denominator by D_x.

By reference to page 27, it will be seen that at age ninety-nine the numerator and denominator are each represented by $v^{99}l_{99}$. The value of A_x is, therefore, in this particular case, equal to unity. It is, however, essential that the particular value of $v^{99}l_{99}$ shall be determined l_{99} is equal to one, because this is the number of persons living at that age according to the Table of Mortality we are now using. The question, then, is simply, what will $0.961538 become when raised to the ninety-ninth power? This multiplication is a matter of simple arithmetic. By referring to page twenty-eight, it will be seen that, at age ninety-eight,

$$A_{98} = \frac{v^{98}l_{98} + v^{99}l_{99}}{v^{98}l_{98}}.$$

Referring to the previous multiplication, we find v to the 98th power.

Multiply this by the number living at age ninety-eight, which, by the Table of Mortality we are now using, is four, and we have the numerical value of $v^{98}l_{98}$. This is the first term of the numerator, and it is at the same time the denominator of the fraction which gives the value of A at the age ninety-eight. Add to this first term of the numerator the value of $v^{99}l_{99}$, just before obtained, and we have the numerator of A at the age ninety-eight. And so working back an age one year less each successive time, we find the first term of the numerator; and this, in every case, gives the denominator of that age. The second term of the numerator will be the first term of the numerator, and also the denominator of an age one year greater; and thus, after finding the first term of the numerator at any age, it not only gives the denominator for that age, but we have only to add this to the numerator of the age one year greater in order to obtain the numerator at the age sought for.

Continuing in this way, diminishing one year at a time, we will finally obtain $A_{10}=\frac{N_{10}}{D_{10}}$; in which the first term of the numerator and the whole denominator will be expressed by $v^{10}l_{10}$. The second term of this numerator is the first term of the numerator of an age one year greater; the third term of the numerator is the first term of the numerator of an age two years greater, and in like manner to age ninety-nine.

$$\text{Or, } A_{10}=\frac{D_{10}+D_{11}+D_{12}+D_{13}+\ldots\text{ to }D_{99}}{D_{10}}.$$

Before giving the following table of numerators and denominators and the resulting values of A_x at the different ages, all of which have been calculated by the above method, only using a table of logarithms to find the different powers of v, instead of actually performing the multiplications indicated; the following extract is given as a matter of interest connected with these tabulated values of the numerators and denominators of the fractions that express the value of A at the different ages. Professor Elizur Wright says, in a foot note, pages 362, 363 of his official reports, published in 1865, alluding to the device of multiplying both the numerator and denominator of A_x by v to the x power: "This happy thought occurred quite independently to two mathematicians, one an erudite Danish or German Professor, and the other an unlearned Englishman, a poor farmer's boy, in fact. John Nicholas Tetens, Professor of Mathematics in the University of Kiel, published the method in 1785. None of the mathematicians or Life Insurance Actuaries of England were the wiser for this, but plodded on, making, in a far more laborious way, the computations demanded in practice, till George Barrett, a wholly self-taught and rather poverty-stricken computer, astonished them about 1811 by a vast mass of tables computed on a method of his own, which was the one above explained. He had commenced his labors on these tables about the time when Tetens published in German, but it is quite certain that he was as ignorant of the existence of that publication, not knowing either German or French, as were all the English Professors. It took the *savans* of the Royal Society several years, after their attention was called to it, to recognize the value of the discovery. But after the death of poor Barrett, in 1821, it soon ranked among the grand *English* discoveries in the field of useful knowledge, under the name of a slight improver, Griffith Davies. A quarter of a century rolled away, and then it was discovered by the scientific Englishman that this wonderful practical discovery had been for sixty-five years in their own libraries, *in German*, without their knowing it! In the standing feud between genius and culture, this is a feather in the cap of the former."

Without further allusion to "THE FAMOUS COLUMNAR METHOD OF TETENS AND BARRETT," we will now refer to the column in TABLE I, headed c_x. This represents the actual risk on one dollar for one year at the age x, and is expressed by $v\,\frac{d_x}{l_x}$. l_x being the number living at the age x, and d_x being the number of deaths between the age x and the age $x+1$. For example: at age forty, $c_x = 0.961538$, multiplied by the number of deaths between the age forty and age forty-one, which, by the Actuaries' Table of Mortality, is 815; and the product divided by the number living at the age forty, according to the same table, this number is 78,653. The result is c_x at age forty, is equal to 0.009963.

The values placed in the column headed u_x are obtained by dividing the *ratio* of interest by UNITY, *minus* the *ratio* of interest, multiplied by c_x. If we call the *ratio* of interest r, then $rv = \$1$, and r is equal to unity, divided by v. When the rate of interest is four per cent., v is equal to \$0.961538, and r is equal to 1.04. The formula which expresses the value of u_x is, $u_x = \frac{r}{1 - rc_x}$.

TABLE II is entirely similar to TABLE I, except in the rate of mortality and the rate of net interest. In TABLE II the American experience rate of mortality, and four and one half per cent. net interest, is taken as the basis of the calculations, instead of the Actuaries' rate of mortality and four per cent. interest, which was assumed as the basis of the calculations in TABLE I. The latter is in conformity with the law of some of the States, and the former of some other States. Many of the States have no laws regulating the rate of mortality or the rate of net interest upon which these calculations are to be made.

TABLE III shows the actual value of the risk on $1,000 for one year at any age, the net single premium to insure $1,000 for whole life at any age, the net annual premium to insure $1,000 for whole life at any age, and the manner of determining the Trust Fund Deposit at the end of any policy year.

In Table IV the rate of mortality is the American experience, and four and one half per cent. interest; otherwise, it is similar to Table III.

None of these are what are technically called VALUATION TABLES. The latter are very voluminous; and besides taking a great deal of time to construct, ARE VERY EXPENSIVE, and only useful in large offices where an immense number of policies have to be promptly valued.

There is reason to believe that a large proportion of the 600,000 policy-holders in this country, whose lives are now insured for more than $2,000,000,000, are nearly as ignorant of the peculiar principles upon which the calculations of important money-values in this business are based, as the writer was but a few months since, when the subject was first brought to his attention. I am satisfied that their interests demand that more of their number should comprehend the nature and peculiarities of the business in which their money is invested, and I have, therefore, endeavored to explain the subject to them. The foregoing Notes were intended, however, to be *suggestive;* it having been no part of my purpose or desire to force conclusions upon the mind of any one. On the contrary, I have endeavored to give to the reader who was not previously acquainted with the theory and principles of this business the means by which to judge for himself; and form his own conclusions in reference to the principles, the data, the formula, and rules used in calculating money values in Life Insurance. I have taken the trouble to write and publish these "Notes," with the hope that they may afford to the uninitiated the assistance I would have been glad to receive when I commenced to investigate this subject. I agree with the author, whose opinion is quoted on the first page of this book, that *Life Insurance, like all good things, prospers in light rather than in darkness.* If further information or better "light" is required than that which the foregoing "Notes" furnish, I respectfully refer all inquirers to the learned writings of authors experienced in this business.

The calculations for the arithmetical examples, introduced for the purpose of illustration in the preceding Notes, were made from printed tables, in the published Reports of the Insurance Commissioner of Massachusetts, Professor Elizur Wright. The manuscript of these Notes was placed in the hands of three gentlemen who were entirely unacquainted with the subject of Life Insurance. After careful reading of the manuscript, they kindly offered to assist me in furnishing an additional arithmetical example for illustration, viz: the tables hereto appended. The labor proved to be less than was anticipated. The four tables annexed were made under my general supervision and direction by Major Henry T. Stanton, and Professors John A. Monroe and William S. Smith, all at present residents of Frankfort, Kentucky.

TABLE I.

Actuaries' Rate of Mortality. $\left\{ v = \frac{100}{104} \right.$.

4 PER CENT.

Age.	Number living.	Number of Deaths.	N_x	D_x	A_x	u_x	c_x
10	100,000	676	1381771.	67556.41	20.453589	1.04708	.006500
11	99,324	674	1314215.	64519.00	20.369426	1.04710	.006525
12	98,650	672	1249696.	61616.50	20.281841	1.04713	.006550
13	97,978	671	1188079.	58843.05	20.190643	1.04717	.006585
14	97,307	671	1129236.	56192.36	20.095900	1.04722	.006630
15	96,636	671	1073044.	53658.53	19.997641	1.04727	.006677
16	95,965	672	1019385.	51236.49	19.895670	1.04733	.006733
17	95,293	673	968148.8	48920.87	19.790097	1.04740	.006791
18	94,620	675	919228.0	46707.08	19.680699	1.04747	.006860
19	93,945	677	872520.9	44590.30	19.567511	1.04755	.006929
20	93,268	680	827930.6	42566.30	19.450379	1.04764	.007010
21	92,588	683	785364.3	40630.72	19.329324	1.04773	.007093
22	91,905	686	744733.5	38779.80	19.204165	1.04782	.007177
23	91,219	690	705953.7	37009.94	19.074705	1.04793	.007273
24	90,529	694	668943.8	35317.30	18.940964	1.04803	.007371
25	89,835	698	633626.5	33698.62	18.802746	1.04814	.007471
26	89,137	703	599927.9	32150.75	18.659844	1.04827	.007583
27	88,434	708	567777.1	30670.37	18.512235	1.04839	.007698
28	87,726	714	537106.7	29254.63	18.359717	1.04854	.007826
29	87,012	720	507852.1	27900.52	18.202238	1.04868	.007956
30	86,292	727	479951.6	26605.42	18.039651	1.04884	.008101
31	85,565	734	453346.2	25366.62	17.871755	1.04900	.008248
32	84,831	742	427979.6	24181.75	17.698454	1.04918	.008410
33	84,089	750	403797.8	23048.31	17.519627	1.04936	.008576
34	83,339	758	380749.5	21964.17	17.335037	1.04955	.008746
35	82,581	767	358785.3	20927.30	17.144314	1.04975	.008931
36	81,814	776	337858.0	19935.51	16.947552	1.04996	.009122
37	81,038	785	317922.5	18986.94	16.744273	1.05017	.009314
38	80,253	795	298935.6	18079.83	16.534203	1.05040	.009525
39	79,458	805	280855.8	17212.24	16.317213	1.05064	.009741
40	78,653	815	263643.5	16382.56	16.092936	1.05089	.009963
41	77,838	826	247261.0	15589.23	15.861014	1.05116	.010204
42	77,012	839	231671.7	14830.58	15.621216	1.05146	.010476
43	76,173	857	216841.2	14104.81	15.373564	1.05183	.010818
44	75,316	881	202736.3	13409.73	15.118590	1.05231	.011247
45	74,435	909	189326.6	12743.15	14.857127	1.05286	.011742
46	73,526	944	176583.5	12103.40	14.589578	1.05353	.012345
47	72,582	981	164480.1	11488.46	14.316984	1.05425	.012996
48	71,601	1,021	152991.6	10897.30	14.039404	1.05504	.013711
49	70,580	1,063	142094.3	10328.75	13.757163	1.05590	.014482
50	69,517	1,108	131765.6	9781.92	13.470321	1.05684	.015326
51	68,409	1,156	121983.7	9255.77	13.179206	1.05788	.016248
52	67,253	1,207	112727.9	8749.40	12.884072	1.05901	.017257
53	66,046	1,261	103978.49	8261.90	12.585301	1.06024	.018359
54	64,785	1,316	95716.59	7792.45	12.283247	1.06156	.019532
55	63,469	1,375	87924.14	7340.54	11.977884	1.06303	.020831

TABLE I—Continued.

Actuaries' Rate of Mortality. $\left\{ v = \frac{100}{104} \right.$.

4 PER CENT.

Age.	Number living.	Number of Deaths.	N_x	D_x	A_x	u_x	c_x
56	62,094	1,432	80583.60	6905.30	11.669819	1.06462	.022237
57	60,658	1,497	73678.30	6486.16	11.359310	1.06632	.023730
58	59,161	1,561	67192.14	6082.77	11.046306	1.06819	.025371
59	57,600	1,627	61109.37	5694.50	10.731297	1.07023	.027160
60	55,973	1,698	55414.87	5320.81	10.414743	1.07254	.029169
61	54,275	1,770	50094.06	4960.96	10.097652	1.07506	.031357
62	52,505	1,844	45133.10	4614.59	9.780500	1.07786	.033770
63	50,661	1,917	40518.51	4281.27	9.464133	1.08090	.036384
64	48,744	1,990	36237.24	3960.84	9.148877	1.08427	.039255
65	46,754	2,061	32276.40	3653.01	8.835565	1.08796	.042386
66	44,693	2,128	28623.39	3357.68	8.524752	1.09199	.045782
67	42,565	2,191	25265.71	3074.81	8.216999	1.09644	.049494
68	40,374	2,246	22190.90	2804.37	7.912971	1.10126	.053490
69	38,128	2,291	19386.53	2546.50	7.613010	1.10648	.057776
70	35,837	2,327	16840.03	2301.43	7.317203	1.11222	.062436
71	33,510	2,351	14538.60	2069.22	7.026126	1.11847	.067460
72	31,159	2,362	12469.38	1850.05	6.740022	1.12530	.072889
73	28,797	2,358	10619.33	1644.04	6.459289	1.13275	.078734
74	26,439	2,339	8975.29	1451.37	6.184012	1.14093	.085065
75	24,100	2,303	7523.92	1272.086	5.914628	1.14988	.091885
76	21,797	2,249	6251.833	1106.274	5.651250	1.15965	.099211
77	19,548	2,179	5145.5595	953.9711	5.392799	1.17047	.107182
78	17,369	2,092	4191.5884	815.0313	5.142853	1.18242	.115812
79	15,277	1,987	3376.5571	689.2936	4.898572	1.19549	.125062
80	13,290	1,866	2687.2635	576.5777	4.660720	1.20987	.135006
81	11,424	1,730	2110.6858	476.5601	4.428990	1.22560	.145611
82	9,694	1,582	1634.1257	388.8384	4.202583	1.24282	.156917
83	8,112	1,427	1245.2873	312.8677	3.980235	1.26200	.169146
84	6,685	1,268	932.4196	247.9139	3.761058	1.28344	.182383
85	5,417	1,111	684.5057	193.1634	3.543653	1.30833	.197207
86	4,306	958	491.3423	147.6409	3.327938	1.33759	.213923
87	3,348	811	343.7014	110.3786	3.113829	1.37246	.232917
88	2,537	673	233.32281	80.42416	2.901187	1.41549	.255071
89	1,864	545	152.89865	56.81705	2.691079	1.46972	.281137
90	1,319	427	96.08160	38.65844	2.485382	1.53785	.311279
91	892	322	57.42316	25.13801	2.284493	1.62751	.347103
92	570	231	32.28515	15.44570	2.090236	1.74867	.389676
93	339	155	16.83945	8.83281	1.906467	1.91609	.439641
94	184	95	8.006643	4.609818	1.736867	2.15011	.496446
95	89	52	3.396825	2.143990	1.584347	2.50162	.501798
96	37	24	1.252835	.857040	1.461816	2.95999	.623701
97	13	9	.395795	.289540	1.366978	3.37999	.665680
98	4	3	.106255	.085663	1.240384	4.15999	.721154
99	1	1	.020592	.020592	1.000000	--------	.961538

Table II.

American Experience Rate of Mortality. $\left\{ v = \frac{100}{104.5} \right.$.

4½ PER CENT.

Age.	Number living.	Number of deaths.	N_x	D_x	A_x	u_x	c_x
10	100,000	749	1214144.06	64392.77	18.855286	1.052885	.007167
11	99,251	746	1149751.29	61158.33	18.799583	1.052914	.007193
12	98,505	743	1088592.96	58084.84	18.741431	1.052942	.007218
13	97,762	740	1030508.12	55164.33	18.680699	1.052971	.007244
14	97,022	737	975343.79	52389.24	18.617252	1.052999	.007269
15	96,285	735	922954.55	49752.43	18.550916	1.053038	.007305
16	95,550	732	873202.12	47246.55	18.481821	1.053066	.007331
17	94,818	729	825955.57	44865.63	18.409536	1.053096	.007357
18	94,089	727	781089.94	42603.53	18.333924	1.053136	.007394
19	93,362	725	738486.41	40453.92	18.255002	1.053177	.007431
20	92,637	723	698032.49	38411.27	18.172596	1.053220	.007469
21	91,914	722	659621.22	36470.31	18.086522	1.053274	.007517
22	91,192	721	623150.91	34625.68	17.996738	1.053328	.007566
23	90,471	720	588525.23	32872.65	17.903180	1.053382	.007615
24	89,751	719	555652.58	31206.73	17.805534	1.053439	.007666
25	89,032	718	524445.85	29623.67	17.703614	1.053496	.007717
26	88,344	718	494822.18	28119.40	17.597187	1.053564	.007779
27	87,596	718	466702.78	26689.75	17.486261	1.053636	.007844
28	86,878	718	440013.03	25331.08	17.370486	1.053709	.007908
29	86,160	719	414681.95	24039.93	17.249715	1.053793	.007985
30	85,441	720	390642.02	22812.75	17.123883	1.053881	.008064
31	84,721	721	367829.27	21646.43	16.992622	1.053975	.008144
32	84,000	723	346182.84	20537.99	16.855734	1.054072	.008236
33	83,277	726	325644.85	19484.42	16.713063	1.054189	.008342
34	82,551	729	306160.43	18482.83	16.564535	1.054311	.008451
35	81,822	732	287677.60	17530.72	16.409932	1.054433	.008561
36	81,090	737	270146.88	16625.74	16.248753	1.054584	.008697
37	80,353	742	253521.14	15765.19	16.081066	1.054740	.008837
38	79,611	749	237755.95	14947.01	15.906540	1.054925	.009003
39	78,862	756	222808.94	14168.77	15.725304	1.055114	.009173
40	78,106	765	208640.17	13428.66	15.536887	1.055337	.009373
41	77,341	774	195211.51	12724.53	15.341389	1.055564	.009577
42	76,567	785	182486.98	12054.73	15.138245	1.055825	.009811
43	75,782	797	170432.25	11417.36	14.927418	1.056107	.010064
44	74,985	812	159014.89	10810.78	14.708902	1.056439	.010362
45	74,173	828	148204.10	10233.23	14.482603	1.056797	.010682
46	73,345	848	137970.87	9683.250	14.248399	1.057223	.011064
47	72,497	870	128287.62	9159.133	14.006573	1.057694	.011484
48	71,627	896	119128.49	8659.541	13.756972	1.058237	.011970
49	70,731	927	110468.94	8182.981	13.499817	1.058877	.012541
50	69,804	962	102285.96	7727.976	13.235924	1.059603	.013188
51	68,842	1,001	94557.98	7293.276	12.965084	1.060419	.013914
52	67,841	1,044	87264.70	6877.731	12.638010	1.061332	.014726

Table II—Continued.

American Experience Rate of Mortality. $\left\{ v = \frac{100}{104.5} \right.$.

4½ PER CENT.

Age.	Number living.	Number of deaths.	N_x	D_x	A_x	u_x	c_x
53	66,797	1,091	80386.97	6480.276	12.404850	1.062351	.015629
54	65,706	1,143	73906.69	6099.936	12.115991	1.063499	.016646
55	64,563	1,199	67806.76	5735.716	11.821859	1.064763	.017771
56	63,364	1,260	62071.043	5386.793	11.522829	1.066202	.019029
57	62,104	1,325	56684.250	5052.321	11.199405	1.067780	.020416
58	60,779	1,394	51631.929	4731.607	10.912104	1.069530	.021948
59	59,385	1,468	46900.322	4424.004	10.601303	1.071486	.023655
60	57,917	1,546	42476.317	4128.846	10.287712	1.073659	.025544
61	56,371	1,628	38347.472	3845.581	9.971829	1.076077	.027636
62	54,743	1,713	34501.891	3573.704	9.654386	1.078756	.029944
63	53,030	1,800	30928.187	3312.801	9.335966	1.081716	.032481
64	51,230	1,889	27615.386	3062.540	9.017152	1.085007	.035285
65	49,341	1,980	24552.846	2822.598	8.698695	1.088686	.038400
66	47,361	2,070	21730.248	2592.660	8.381449	1.092761	.041825
67	45,291	2,158	19137.588	2372.577	8.066151	1.097282	.045595
68	43,133	2,243	16765.011	2162.231	7.753570	1.102322	.049762
69	40,890	2,321	14602.780	1961.521	7.444624	1.107884	.054317
70	38,569	2,391	12641.259	1770.508	7.139897	1.114064	.059323
71	36,178	2,448	10870.751	1589.234	6.840261	1.120841	.064751
72	33,730	2,487	9282.5174	1417.892	6.546001	1.128183	.070557
73	31,243	2,505	7863.6254	1256.792	6.256916	1.136089	.076725
74	28,738	2,501	6606.8334	1106.244	5.972338	1.144613	.083280
75	26,237	2,476	5500.5894	966.4789	5.691364	1.153892	.090306
76	23,761	2,431	4534.1105	837.5806	5.413345	1.164099	.097905
77	21,330	2,369	3696.5299	719.5095	5.137566	1.175562	.106281
78	18,961	2,291	2977.0204	612.0552	4.863860	1.188616	.115623
79	16,670	2,196	2364.9652	514.9305	4.592799	1.203546	.126060
80	14,474	2,091	1850.0347	427.8438	4.324116	1.221459	.138245
81	12,383	1,964	1422.1909	350.2727	4.060268	1.241852	.151689
82	10,419	1,816	1071.9182	282.0267	3.800730	1.265587	.166791
83	8,603	1,648	789.89158	222.8423	3.544611	1.292614	.183312
84	6,955	1,470	567.04928	172.3966	3.289214	1.325063	.202257
85	5,485	1,292	394.65268	130.1043	3.033356	1.366999	.225408
86	4,193	1,114	264.54838	95.17522	2.779593	1.423087	.254240
87	3,079	933	169.37316	66.87942	2.532516	1.499325	.289971
88	2,146	744	102.49372	44.60631	2.297740	1.599550	.331762
89	1,402	555	57.887413	27.88676	2.075802	1.729631	.378780
90	847	385	30.000650	16.12194	1.860858	1.915832	.434972
91	462	246	13.878713	8.415103	1.649262	2.235137	.509538
92	216	137	5.463610	3.764914	1.451184	2.857209	.606946
93	79	58	1.698696	1.317686	1.289151	3.931191	.702562
94	21	18	.381010	.335188	1.136706	7.244979	.820232
95	3	3	.045822	.045822	1.000000		.956938

Table III.—*Whole Life Policy for* $1,000—*Actuaries' Rate of Mortality.* $\left\{ v = \frac{100}{104} \right.$. 4 PER CENT.

Age.	Net Amount that will Insure $1,000 for one year at different ages. $v\frac{d_x}{l_x}\times 1000$.	Net Annual Premium that will Insure $1,000 for Life at different ages. $\left(\frac{1}{A_x}-(1-v)\right)\times 1000$	Net Single Premium that will Insure $1,000 for Life at different ages. $\left(1-(1-v)A_x\right)\times 1000$	Trust Fund Deposit or "Reserve" at the end of any Policy Year, of a Whole Life Policy, taken out at different ages.
10	6.50	10.429	213.323	
11	6.52	10.632	216.560	
12	6.55	10.842	219.929	
13	6.58	11.065	223.438	
14	6.63	11.299	227.080	
15	6.68	11.544	230.860	
16	6.73	11.800	234.746	
17	6.79	12.067	238.842	
18	6.86	12.349	243.040	
19	6.93	12.643	247.403	
20	7.01	12.950	251.908	
21	7.09	13.272	256.565	
22	7.18	13.610	261.378	
23	7.27	13.964	266.358	
24	7.37	14.334	271.501	
25	7.47	14.721	276.817	
26	7.58	15.129	282.314	
27	7.70	15.557	287.992	
28	7.83	16.005	293.857	
29	7.96	16.476	299.914	
30	8.10	16.972	306.169	
31	8.25	17.492	312.614	
32	8.41	18.039	319.290	
33	8.58	18.617	326.168	
34	8.75	19.225	333.268	
35	8.93	19.866	340.600	
36	9.12	20.544	348.171	
37	9.31	21.260	355.989	
38	9.52	22.019	364.069	
39	9.74	22.823	372.415	
40	9.96	23.677	381.040	
41	10.20	24.585	389.961	
42	10.48	25.554	399.184	
43	10.82	26.585	408.709	
44	11.25	27.682	418.516	
45	11.74	28.846	428.572	
46	12.34	30.080	438.862	
47	13.00	31.385	449.347	
48	13.71	32.766	460.023	
49	14.48	34.228	470.878	
50	15.33	35.775	481.910	
51	16.25	37.415	493.107	
52	17.26	39.153	504.459	
53	18.36	40.996	515.949	
54	19.53	42.949	527.567	
55	20.83	45.025	539.312	

When insurance is paid for year by year, as indicated in the column of premiums headed

$$v\frac{d_x}{l_x}\times 1000,$$

there is no Deposit on hand at the end of any year, the amount paid each year being just sufficient to pay cost of insurance during that year.

When insurance for whole life is paid for by a net single premium, paid at the age x, the Deposit at the end of any number of years n must be equal to the net single premium at the age $x+n$. When insurance at the age x for whole life is paid for by equal annual premiums, the Deposit that must be on hand at the end of n years may be obtained by taking the difference between the net annual premium at the age $x+n$ and the net annual premium at the age x, and multiplying this difference by the value of large A at the age $x+n$, which, in this case, is obtained from Table I.

TABLE III—Continued.—*Whole Life Policy for* $1,000—*Actuaries' Rate of Mortality.* $\left\{ v = \frac{100}{104} \right.$. 4 PER CENT.

Age.	Net Amount that will Insure $1,000 for one year at different ages. $v \frac{d_x}{l_x} \times 1000.$	Net Annual Premium that will Insure $1,000 for Life at different ages. $\left(\frac{1}{A_x} - (1-v)\right) \times 1000$	Net Single Premium that will Insure $1,000 for Life at different ages. $\left(1-(1-v)A_x\right) \times 1000$	Trust Fund Deposit or "Reserve" at the end of any Policy Year, of a Whole Life Policy, taken out at different ages.
56	22.24	47.229	551.161	When insurance is paid for year by year, as indicated in the column of premiums headed $v \frac{d_x}{l_x} \times 1000$, there is no Deposit on hand at the end of any year, the amount paid each year being just sufficient to pay cost of insurance during that year. When insurance for whole life is paid for by a net single premium, paid at the age x, the Deposit at the end of any number of years n must be equal to the net single premium at the age $x+n$. When insurance at the age x for whole life is paid for by equal annual premiums, the Deposit that must be on hand at the end of n years may be obtained by taking the difference between the net annual premium at the age $x+n$ and the net annual premium at the age x, and multiplying this difference by the value of large A at the age $x+n$, which, in this case, is obtained from Table I.
57	23.73	49.571	563.103	
58	25.37	52.066	575.142	
59	27.16	54.723	587.258	
60	29.17	57.556	599.430	
61	31.36	60.572	611.628	
62	33.77	63.778	623.826	
63	36.38	67.198	635.995	
64	39.25	70.838	648.120	
65	42.39	74.718	660.170	
66	45.78	78.848	672.125	
67	49.49	83.238	683.970	
68	53.49	87.918	695.650	
69	57.78	92.898	707.192	
70	62.44	98.198	718.569	
71	67.46	103.868	729.764	
72	72.89	109.898	740.762	
73	78.73	116.358	751.566	
74	85.06	123.238	762.152	
75	91.88	130.608	772.514	
76	99.21	137.488	782.644	
77	107.18	146.938	792.546	
78	115.81	155.988	802.198	
79	125.06	165.678	811.593	
80	135.01	176.098	820.742	
81	145.61	187.318	829.654	
82	156.92	199.478	838.362	
83	169.15	212.778	846.914	
84	182.38	227.428	855.344	
85	197.21	243.738	863.707	
86	213.92	262.104	872.002	
87	232.92	282.687	880.237	
88	255.07	306.226	888.417	
89	281.14	333.138	896.497	
90	311.28	363.889	904.408	
91	347.10	399.304	912.142	
92	389.68	439.948	919.606	
93	439.64	486.067	926.675	
94	496.45	537.288	933.198	
95	501.80	592.209	939.064	
96	623.70	645.618	943.777	
97	665.68	693.078	947.424	
98	721.15	767.738	952.293	
99	961.54	961.538	961.538	

Table IV.

Whole Life Policy for $1,000—American Experience Rate of Mortality. $\left\{ v=\frac{100}{104.5}\right.$.

4½ PER CENT.

Age.	Net Amount that will Insure $1,000 for one year at different ages. $v\frac{d_x}{l_x}\times 1000.$	Net Single Premium that will Insure $1,000 for Life at different ages. $\left(1-(1-v)A_x\right)\times 1000$	Net Annual Premium that will Insure $1,000 for Life at different ages. $\left(\frac{1}{A_x}-(1-v)\right)\times 1000$
10	7.16	188.050	9.972
11	7.19	190.450	10.130
12	7.21	192.953	10.295
13	7.24	195.568	10.469
14	7.27	198.300	10.652
15	7.30	201.157	10.843
16	7.33	204.132	11.045
17	7.36	207.245	11.257
18	7.39	210.501	11.481
19	7.43	213.899	11.717
20	7.47	217.448	11.966
21	7.52	221.155	12.227
22	7.57	225.021	12.503
23	7.61	229.050	12.794
24	7.67	233.255	13.100
25	7.72	237.643	13.423
26	7.78	242.226	13.765
27	7.84	247.003	14.125
28	7.91	251.989	14.506
29	7.98	257.191	14.910
30	8.06	262.608	15.336
31	8.14	268.260	15.787
32	8.24	274.155	16.265
33	8.34	280.299	16.771
34	8.45	286.695	17.308
35	8.56	293.352	17.876
36	8.70	300.295	18.481
37	8.84	307.514	19.123
38	9.00	315.429	19.805
39	9.17	322.832	20.529
40	9.37	330.948	21.301
41	9.58	339.366	22.121
42	9.81	348.114	22.996
43	10.06	357.193	23.929
44	10.36	366.602	24.924
45	10.68	376.346	25.986
46	11.06	386.433	27.121
47	11.48	396.846	28.333
48	11.97	407.594	29.628
49	12.54	418.668	31.013
50	13.18	430.032	32.490
51	13.91	441.695	34.068
52	14.73	453.626	35.753

Trust Fund Deposit or "Reserve" at the end of any Policy Year, of a Whole Life Policy taken out at different ages.

When insurance is paid for year by year, as indicated in the column of premiums headed

$$v\frac{d_x}{l_x}\times 1000,$$

there is no Deposit on hand at the end of any year, the amount paid each year being just sufficient to pay cost of insurance during that year. When insurance for whole life is paid for by a net single premium, paid at the age x, the Deposit at the end of any number of years n must be equal to the net single premium at the age $x+n$. When insurance at the age x for whole life is paid for by equal annual premiums, the Deposit that must be on hand at the end of n years may be obtained by taking the difference between the net annual premium at the age $x+n$ and the net annual premium at the age x, and multiplying this difference by the value of large A at the age $x+n$, which, in this case, is obtained from Table II.

Table IV—Continued.

Whole Life Policy for $1,000—American Experience Rate of Mortality. $\left\{ v = \frac{100}{104.5} \right.$.

4½ PER CENT.

Age.	Net amount that will Insure $1,000 for one year at different ages. $v\frac{d_x}{l_x}\times 100$.	Net Single Premium that will Insure $1,000 for Life at different ages. $\left(1-(1-v)A_x\right)\times 1000$	Net Annual Premium that will Insure $1,000 for Life at different ages. $\left(\frac{1}{A_x}-(1-v)\right)\times 1000$
53	15.63	465.820	37.551
54	16.65	478.259	39.473
55	17.77	490.925	41.527
56	19.03	503.801	43.722
57	20.42	516.866	46.069
58	21.95	530.099	48.579
59	23.66	543.484	51.265
60	25.54	556.989	54.141
61	27.64	570.591	57.122
62	29.94	584.261	60.517
63	32.48	597.973	64.050
64	35.28	611.702	67.837
65	38.40	625.415	71.897
66	41.82	639.076	76.249
67	45.59	652.654	80.913
68	49.76	666.114	85.910
69	54.31	679.418	91.263
70	59.32	692.541	96.996
71	64.75	705.443	103.131
72	70.56	718.115	109.703
73	76.72	730.564	116.761
74	83.28	742.818	124.377
75	90.31	754.917	132.643
76	97.90	766.889	141.667
77	106.28	778.765	151.583
78	115.62	790.551	162.536
79	126.06	802.224	174.670
80	138.24	813.794	188.199
81	151.69	825.156	203.227
82	166.79	836.332	220.045
83	183.31	847.361	239.056
84	202.26	858.359	260.962
85	225.41	869.377	286.606
86	254.24	880.305	316.703
87	289.97	890.944	351.802
88	331.76	901.543	392.148
89	378.78	910.609	438.668
90	434.97	919.868	494.324
91	509.54	929.980	563.270
92	606.95	937.509	646.031
93	702.56	944.846	732.643
94	820.23	951.051	836.673
95	956.94	956.938	956.938

Trust Fund Deposit or "Reserve" at the end of any Policy Year, of a Whole Life Policy, taken out at different ages.

When insurance is paid for year by year, as indicated in the column of premiums headed

$$v\frac{d_x}{l_x}\times 1000,$$

there is no Deposit on hand at the end of any year, the amount paid each year being just sufficient to pay cost of insurance during that year.

When insurance for whole life is paid for by a net single premium, paid at the age x, the Deposit at the end of any number of years n must be equal to the net single premium at the age $x+n$. When insurance at the age x for whole life is paid for by equal annual premiums, the Deposit that must be on hand at the end of n years may be obtained by taking the difference between the net annual premium at the age $x+n$ and the net annual premium at the age x, and multiplying this difference by the value of large A at the age $x+n$, which, in this case, is obtained from Table II.

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SECOND LESSON IN LIFE INSURANCE CALCULATIONS.

To determine the amount of money that will, at a given rate of interest, produce $1 in any given number of years, the interest being compounded annually: raise the expression that gives the amount that will produce $1 in one year, at the same rate of interest, to a power indicated by the number of years. If the rate of interest is 4 per cent., the amount of money that will produce $1, in one year, is 100 divided by 104; which is equal to the decimal $0.96153846153846 (and the series of decimal figures 153846 will be repeated to infinity). If we multiply $0.96153846 by 0.96153846, we obtain the amount that will, if invested at 4 per cent. compound interest, produce $1 in two years. Multiply this result by 0.96153846, and we obtain the amount that will, if invested for three years at 4 per cent. compound interest, produce $1 at the end of that time; and so on for other years.

Suppose that the rate of interest is 4½ per cent.; that the person to be insured is 20 years old; and that he desires to have $1 insured, to be paid to his heirs at the end of 30 years, provided he dies between age 49 and age 50. How much money in hand, at age 20, will be required to pay for the proposed insurance? Divide 100 by 104½; then raise this quantity to the 30th power. This gives the amount that will, at 4½ per cent. compound interest, produce $1 in 30 years. From the Mortality Table obtain the number of deaths between age 49 and age 50: it is 927. Divide this by the number living at age 20; this number living is 92,637. Then $\frac{927}{92{,}637}$ is the fraction that represents, at age 20, the chance or probability at that time that the insured will die between the age 49 and age 50. Multiply the amount that will produce $1 in 30 years by the fraction which represents the chance that the insured will die during the 30th year from that time, and we obtain the amount that will insure $1, to be paid to the heirs of the insured at the end of 30 years, in case he dies between 49 and 50 years of age. In like manner the calculation is made at any age, for insurance during any named year. At any age, if the calculation is made for insurance during every year from that age to the limit of the Table, the sum of all these respective yearly amounts will, at that age, effect the insurance for whole life.

FIRST LESSON IN LIFE INSURANCE CALCULATIONS.

To obtain at any age the amount that will insure $1,000 to be paid to the heirs of the insured at the end of one year, in case the insured dies during the year: a Table showing the Rate of Mortality must be furnished, and a rate of interest fixed upon. Assume that the Table is that which purports to give the rate of mortality among insured lives in this country, which is called

American Experience Rate of Mortality.

Age.	Number living.	Number of deaths.	Age.	Number living.	Number of deaths.	Age.	Number living.	Number of deaths.	Age.	Number living.	Number of deaths.
10	100,000	749	32	84,000	723	54	65,706	1,143	76	23,761	2,431
11	99,251	746	33	83,277	726	55	64,563	1,199	77	21,330	2,369
12	98,505	743	34	82,551	729	56	63,364	1,260	78	18,961	2,291
13	97,762	740	35	81,822	732	57	62,104	1,325	79	16,670	2,196
14	97,022	737	36	81,090	737	58	60,779	1,394	80	14,474	2,091
15	96,285	735	37	80,353	742	59	59,385	1,468	81	12,383	1,964
16	95,550	732	38	79,611	749	60	57,917	1,546	82	10,419	1,816
17	94,818	729	39	78,862	756	61	56,371	1,628	83	8,603	1,648
18	94,089	727	40	78,106	765	62	54,743	1,713	84	6,955	1,470
19	93,362	725	41	77,341	774	63	53,030	1,800	85	5,485	1,292
20	92,637	723	42	76,567	785	64	51,230	1,889	86	4,193	1,114
21	91,914	722	43	75,782	797	65	49,341	1,980	87	3,079	933
22	91,192	721	44	74,985	812	66	47,361	2,070	88	2,146	744
23	90,471	720	45	74,173	828	67	45,291	2,158	89	1,402	555
24	89,751	719	46	73,345	848	68	43,133	2 243	90	847	385
25	89,032	718	47	72,497	870	69	40,890	2,321	91	462	246
26	88,344	718	48	71,627	896	70	38,569	2,391	92	216	137
27	87,596	718	49	70,731	927	71	36,178	2,448	93	79	58
28	86,878	718	50	69,804	962	72	33,730	2,487	94	21	18
29	86,160	719	51	68,842	1,001	73	31,243	2,505	95	3	3
30	85,441	720	52	67,841	1,044	74	28,738	2,501			
31	84,721	721	53	66,797	1,091	75	26,237	2,476			

After having assumed a rate of interest, we obtain the amount that will, at this rate, produce $1 in one year, by dividing 100 by 100, plus the rate of interest. Suppose the interest is assumed to be seven per cent., and that the person to be insured for one year is aged 50. The amount that will, if paid in advance, and invested at 7 per cent., produce $1 certain in one year, when principal and interest at this rate for one year are added together, is obtained by dividing 100 by 107. This makes $0.934579. Then multiply this amount by the number of deaths given in the Table opposite to age 50, which is 962, and divide the product by the number living at the same age, which is 69,804. The result is $0.012879. This is the amount that will insure $1 for one year, if paid in hand at age 50. One thousand times this amount, or $12.88, will insure $1,000 for one year at the same age. In a precisely similar manner, the calculations are made for insurance for one year at any age, and for any amount, and at any rate of interest.

See inside page of cover.