The Life Insurance Actuary and his Mathematics
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Source: *The American Mathematical Monthly*, Vol. 36, No. 6 (Jun. - Jul., 1929), pp. 312-319
Published by: Taylor & Francis, Ltd. on behalf of the Mathematical Association of America
Stable URL: http://www.jstor.org/stable/2298295
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6. *Concluding Remarks*

A great many other cryptographic constructions can, of course, be derived from the algebra, by no means fully developed in this paper, of the bi-operational alphabet. The purpose of the paper, however, will have been accomplished if the single construction described serves to emphasize sufficiently the circumstance that sets which fail to possess in full the character of algebraic fields may still admit a large measure of amusing, and possibly useful, algebraic manipulation. It need hardly be said that if full-fledged finite algebraic fields are employed, the opportunities of the cryptographer are greatly extended; he then has at his disposal a perfectly smooth algebra and its associated geometries. The writer hopes to submit a further communication on this subject. But the number of marks in a finite field is necessarily either a prime or a power of a prime. If our alphabet is to be converted into a finite field, the best that can be done is to omit one letter, say j, to obtain a field of twenty-five marks; or to adjoin an additional symbol so that a field of twenty-seven marks is available. The bi-operational alphabet[1] of twenty-six letters, and the further development of its algebra, should therefore be of some importance in cryptography.

If polygraphic ciphers based upon normal transformations (linear ciphers) prove to be of real interest, we shall indicate a surprising way in which these ciphers may be manipulated easily and quickly, even for fairly large values of n(say $n=8$, 9, or 10), and thus made effective in a distinctly practical sense. It should be remarked that a cipher of type C_n in which $n>4$, although easy to use, is extraordinarily difficult to "break," offering very high resistance to the methods of cryptanalysis.

THE LIFE INSURANCE ACTUARY AND HIS MATHEMATICS[2]

By RAYMOND V. CARPENTER, Metropolitan Life Insurance Co.

It is estimated that the amount of life insurance in force in United States companies at the end of 1928 is about $95,000,000,000. The assets are about $16,000,000,000 and the premium collections in 1928 were over $3,000,000,000.

The employed personnel of a life insurance company consist of the field or agency force and the home office force. An important part of the home office force is the actuarial department.

The actuary has a wide range of duties. He must be reasonably familiar with the work of all departments of the company, and is sometimes called its "technical" man. His two main duties are, first, the calculation of the premiums

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[1] The bi-operational alphabet employed in this paper is an example of a "ring." See the Bulletin of the National Research Council, *Report on Algebraic Numbers*, p. 59.

[2] This paper was read by invitation before the Mathematical Association of America at New York City on Dec. 29, 1928.

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charged, according to plan of insurance and age, and second, the calculation of the reserves that must be held by the company to provide, with the aid of future premiums and interest earnings, for future obligations, which reserves must be covered by safe interest-earning assets. The $95,000,000,000 of insurance in force is mainly the result of solicitation by the agency forces, but for the sufficiency of the $3,000,000,000 yearly premiums and of the $16,000,000,000 of assets the actuaries are mainly responsible.

Bases of Calculation of Premiums and Reserves

In the calculation of premiums and reserves, the actuary must deal mainly with two factors, namely, the rates of mortality (death rates) according to age and the rate of compound interest. He must assume in his calculations a mortality table showing death rates which it seems safe to assume will not be exceeded and a rate of interest (usually 3 or $3\frac{1}{2}$ per cent.) on which he can safely rely. The necessity for conservatism in selecting these standards will be realized when it is considered that once a policy is issued, the premium is fixed for all time—perhaps for fifty years or more—and cannot be increased, even though it prove insufficient. In this respect life insurance differs from the usual commercial enterprise, where contracts are of short durations and prices can readily be changed.

A third factor entering into the calculation of premiums is that of management expenses and commissions. It is taken care of, however, by an additional charge termed the "loading," the calculation of which is usually relatively simple. The premium inclusive of loading is called the "gross" premium; exclusive of loading, the "net" premium.

Mortality and compound interest, then, are the two factors upon which most of the calculations in actuarial science depend. The combination of these two factors to meet various contingencies results in innumerable formulae, which become fairly complex when the probabilities of survivorship of more than one life are involved, as frequently happens.

Efforts to Discover A "Law of Mortality"

Various attempts have been made to discover a definite law of mortality. Benjamin Gompertz, about one hundred years ago, proposed "that death may be the consequence of two generally coexisting causes; the one, chance, without previous disposition to death or deterioration; the other, a deterioration, or increased inability to withstand destruction." In 1860, Makeham developed this theory to the point of obtaining the expression $l_x = ks^x(g)^{c^x}$, where x is the age, and l the number of lives in a given group of entrants which survive to age x, the other four symbols being constants. Makeham's law applies, with remarkable closeness, to many mortality tables from about age 20 to the end of life, and while it is not considered a true law of mortality, it forms a favorite basis for graduation of mortality tables, for the two reasons, first, that it affords a satisfactory method for continuing a mortality table through the late ages of life, where actual data are meagre, and second, that the special

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properties of a table so graduated make it readily adaptable to the calcuation of many life insurance benefits involving more than one life. In 1924, W. P. Elderton presented a paper at the International Mathematical Congress at Toronto making certain suggestions as to the application of frequency curves to mortality experiences, but did not arrive at a definite "law of mortality."

It is usually conceded, then, that there is no true "law of mortality," and actuaries are generally agreed that the only way to obtain a reliable mortality table is to construct it from the actual record of the deaths and the number living among persons of the class to which the mortality table is applicable. For it must be remembered that mortality rates differ among different classes of persons. For example, persons insured under policies known as "industrial," which are for relatively small amounts, with premiums payable weekly, experience much higher rates of mortality than persons whose better economic conditions permit them to insure under so-called "ordinary" policies for higher amounts. Mortality tables based on insured lives differ from those based upon general population data. The selection of a proper mortality table to apply to a given class of risks is one of the problems of the actuary.

Elementary Principles

The more elementary calculations of actuarial science are set forth in several text books, such as the late Professor Dowling's *Mathematics of Life Insurance*. The more advanced calculations may be found in the two standard text-books—both in English—which are in universal use among English-speaking actuarial students, namely, Spurgeon's *Life Contingencies*, and *Actuarial Theory* by Robertson and Ross, and in various publications of the actuarial societies of America and Great Britain. For the study of compound interest, the (British) Institute of Actuaries' Text Book, Part I, by Todhunter, is a standard.

Time will not permit a discussion of the formulae used for the calculation of premiums. It will be sufficient to say that if we assume for convenience that premiums are payable at the beginning of each year and claims at the end of the year (which assumptions can be appropriately modified), and if we let v equal the present value of 1 due in one year, then the net single premium for a life aged x for insurance of 1 for the whole of life is

$$(vd_x + v^2d_{x+1} + \cdots \text{ to end of table})/l_x\,;$$

that the present value of an annuity of 1 payable at the beginning of each year throughout life is

$$(l_x + vl_{x+1} + v^2l_{x+2} + \cdots \text{ to end of table})/l_x\,;$$

and that the net annual premium is the net single premium divided by the present value of the annuity. This general principle, suitably modified, underlies the calculation of premiums and annuity values generally.

It might also be mentioned that by the simple algebraic device of multi-

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plying numerator and denominator in the foregoing expressions by v^x we obtain a series of terms in which the exponents of v bear a constant relation, at all ages, to the subscripts of d and l, thus enabling us to avoid the large number of combinations that would otherwise occur, and to construct so-called "commutation columns" which greatly facilitate life insurance calculations.

"Select" Mortality Tables

Because of careful selection, medical or otherwise, insured persons usually show especially favorable mortality during the first few years of the policy. Hence so-called "select" tables are sometimes used. These consist of several "l" columns—the first column representing the number of persons entering the table at each age at the beginning of the insurance, the second column the number of them surviving one year, and so on. At the end of the "select" period, say five years, the various l's merge into a common "ultimate" table.

Reserves

The annual rate of mortality is very high in the first year of life. Then it decreases rapidly until about age 12. Thereafter it increases until it becomes very high at the older ages. If a group of persons each aged twenty became insured on the plan of simply paying the current mortality cost each year the premiums would be very low at first, but would increase each year, and in the later years would be very high. For this reason people prefer to pay premiums which will remain *level* throughout the period of the policy, except as the cost may be reduced by dividends. It is the level premium plan which, in the main, gives rise to the very large *reserves* that must be carried by life insurance companies and therefore to the huge volume of assets. Under the level premium plan, it is evident that in the early years of the policy the premiums largely exceed the current mortality cost, and that this excess will be needed in later years when the mortality costs exceed the premiums. The excess, with interest accretions, must therefore be held as a "reserve." Endowment features and plans under which payment of premiums is limited to a specified number of years, naturally require larger reserves than whole life policies. The laws of the several States fix the standards of mortality and interest on which the reserves must be based.

Disability Benefits

In recent years life insurance companies have been offering policies which provide not only for payment of insurance at death, but for certain benefits in event of the so-called "total and permanent disability" of the person insured. A common disability benefit is the waiver of the premiums and payment of a monthly income during the continuance of disability, without deduction from the life insurance benefit. In computing premiums and reserves under such policies the actuary meets added complications, for he must base his calculations upon a combined mortality and disability table, showing for each age (1) the

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rate of mortality among *active* lives (i.e., those not disabled), (2) the rate of disability, and (3) the rate of mortality among "disabled" lives. If the disability benefit is such that recoveries from disability are frequent, then adjustment is needed for the recoveries.

Actuarial Societies

Naturally, actuaries have formed societies for the purposes of better acquaintance with one another and the interchange of ideas. The principal societies in Great Britain and America are the Institute of Actuaries of Great Britain, organized in 1848, the Faculty of Actuaries in Scotland (1856), the Actuarial Society of America (1889) and the American Institute of Actuaries (1909), the last being rather more closely identified with the western and southern companies, than the Actuarial Society, although its membership also includes many eastern actuaries. Many actuaries are members of both of the American societies. Both have many members from Canada. There is also the Casualty Actuarial Society, in America, which, however, is associated with casualty insurance more than life insurance.

These societies hold regular meetings for the presentation and discussion of papers, which are printed in their published transactions, and for the informal discussion of topics of current interest. Through committees they cooperate in compiling, from the joint experience of the various companies, tables of rates of mortality for standard lives or for various special classes of lives, especially those showing certain impairments. Occasionally they publish text books or reports on subjects of current interest. They hold annual examinations open to properly qualified actuarial students, and the passage of these examinations entitles the candidates to membership in the society. The great majority of present members have entered through examination.

Actuarial Examinations

The success of the actuarial student in his work, up to a certain point, is largely dependent upon passage of prescribed examinations, of which a brief description may be of interest. The subjects in the two American societies are quite similar—in fact, the examinations of the first two days will be identical next year. It will therefore be sufficient to treat of the Actuarial Society.

There are two grades of membership, Associateship and Fellowship. Candidates must be proposed by a Fellow and approved by the Society. The examinations are held in April, at a number of centers convenient for the candidates. The examinations for Associateship occupy four days—that is, two each year—but not more than two days' examinations may be taken in one single year. The examinations for Fellowship consist of two parts, and one or both may be taken in a single year. One can become an Associate, therefore, in a minimum of two years and a Fellow in three. Rarely, however, do students pass all of the examinations within the three years.

The first day's examinations for Associateship embrace arithmetic, elementary algebra, plane geometry, plane and spherical trigonometry and plane ana-

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lytical geometry. The subjects for the second day are advanced algebra and the elements of the theory of probabilities, differential and integral calculus, the calculus of finite differences, and statistics. Thus far life insurance itself has not been touched. The third days's examinations include the subjects compound interest and annuities certain, the mortality table and its application, and the theory of life contingencies for one life only, including calculation of net premiums and reserves. The fourth day's examinations—the final for Associateship—cover the theory of life contingencies for more than one life, the use of tables involving more than one decrement, such as death and disability, calculations relating to such accident and disability benefits as are included in life policies, construction of actuarial tables, general nature of life insurance and annuity contracts, including statutory requirements, and the history of life insurance.

The examinations for Fellowship cover a wide range of subjects, both theoretical and practical. The subjects include the principles to be observed in making mortality and disability investigations and the methods of constructing and graduating such tables; the sources and characteristics of the principal mortality and disability tables and investigations; selection of risks and premiums for extra hazards, calculation of gross premiums; valuation of the assets and liabilities of life insurance companies; non-forfeiture values and changes in life insurance contracts; analysis and distribution of surplus; investment of life insurance funds; elements of banking and finance; insurance law; pension funds; general questions involving the application of actuarial principles; and current topics of actuarial interest.

To guide students in their studies, each of the American societies has an "Educational Committee," which publishes, for the guidance of students, a suggested course of reading in preparation for each of the examinations. Students and others interested can obtain information relative to actuarial examinations from the Secretary of the Actuarial Society at 256 Broadway New York City, or from the Secretary of the American Institute of Actuaries at 720 North Michigan Avenue, Chicago, Illinois.

Practical Work of the Actuary

The work of the actuary can be partially judged from the subjects of the examinations. His most important duty, as previously stated, is the calculation of premiums and reserves. The solvency of the company, at present and in the distant future, is largely dependent upon him. He has a large part in the preparation of the annual statements required by the Insurance Departments of the several States, including the highly analytical "gain and loss" exhibits, which analyze the gain or loss in surplus from such sources as mortality, interest, expense provisions, etc. In a mutual company, he assists in determining the total amount of so-called dividends (more properly, savings) which can be returned each year to policyholders and calculates the amounts that can equitably be returned to *each class* of policyholders, according to age, plan of insurance, and

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duration of policy, using formulae which aim to allot the dividends in proportion to the share of surplus contributed by the respective classes. He calculates practically all of the data appearing in the rate book, including not only premiums but cash surrender and other non-forfeiture values. He fixes the terms under which policies may be changed from one form to another or otherwise adjusted. He cooperates with the legal department in the drafting of policy forms and in keeping in touch with current legislation; with the medical directors in fixing rules for the acceptance of various classes of risks involving additional risk by reason of occupation or impaired conditions of health, and with the agency department in fixing the compensation of agents. The investment department may call upon him for calculations relating to securities of a complicated or unusual nature. He carefully watches the mortality and disability rates experienced by his company. Either in his own company, or in cooperation with actuaries of other companies, he occasionally conducts mortality investigations relating to special classes of risks, or engages in the construction and graduation of new mortality tables. He may be called upon to devise plans for pensions. He must keep in touch with current developments in insurance. The list of his activities might be extended almost indefinitely, for he is constantly considering new problems of very great variety which arise in the business. On the whole he and his assistants are kept fairly busy. Most of his time is taken up in work of a very practical nature, and it must be confessed that he must often rely upon his assistants for the more complicated and laborious mathematical calculations required.

In most of their everyday work the actuary and his assistants employ no mathematics beyond relatively simple algebra and its application to insurance. Sometimes, however, as in complicated problems requiring the use of summation or interpolation formulae or in graduation, more advanced mathematics are needed. Always, however, the actuaries must have the capacity for analysis and must have an instinctive sense of proportion.

Preparation for Examinations

While most men with actuarial training are associated with life insurance companies, there are other opportunities. The various State Insurance Departments usually employ one or more actuarially trained men. Some actuaries, not connected wth any company, open offices of their own or enter into partnership as "consulting actuaries." Women are gradually entering the actuarial field.

Most of the preparation for actuarial examinations is done by the candidates while employed in company or other insurance work. Employees of the actuarial departments of the companies are increasingly being recruited from universities and colleges, efforts being made to select students with rather more than average mathematical talent. The man, however, who is purely a mathematical genius, and is not practical, will not develop into a successful actuary. Most of the students entering the companies have not prepared specifically for the actuarial examinations prior to graduation, but their mathematical

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groundwork is helpful, and in fact almost essential. A number of students do undertake the early examinations while in college, and some are successful.

Actuarial Science in Universities and Colleges

While many universities and colleges offer insurance courses of various kinds, those making a serious effort to train students in actuarial science are few. A tabulation of the candidates who have recently passed the early examinations of the Actuarial Society and the American Institute of Actuaries while university students indicates that there are two universities in Canada, namely, Toronto and Manitoba, and two in the Middle West, namely, Iowa and Michigan, which give serious attention to actuarial training, but none in the Eastern States. Columbia University, however, is successfully conducting correspondence courses leading to the actuarial examinations, most of those enrolled being employees of insurance companies.

Opportunities

In the United States the university or college graduate is paid something like \$1500 a year when he first enters a company for actuarial work. By the time he passes the Fellowship examinations, if he has the other qualifications needed, he should probably be able to command \$3000 or \$4000 yearly. After that his progress depends upon his individual ability and his opportunities.

For a number of years in this country, the demand for capable men of actuarial training and practical experience has exceeded the supply. This is not so true in Canada, and in England and Scotland the case is the reverse. As a result, many of the actuarial officers of life insurance companies in the United States, including some of the leaders in actuarial achievement, have come from England, Scotland, and Canada.

QUESTIONS AND DISCUSSIONS

EDITED BY H. E. BUCHANAN, Tulane University, New Orleans, La.

The department of Questions and Discussions in the Monthly is open to all forms of activity in collegiate mathematics, including the teaching of mathematics, except for specific problems, especially new problems, which are reserved for the department of Problems and Solutions.

DISCUSSIONS

I. ON THE RESOLUTION OF A FRACTION INTO PARTIAL FRACTIONS

By W. C. BRENKE, University of Nebraska

1. In this paper is explained a simple routine for obtaining the undetermined coefficients required to express a given fraction as a sum of partial fractions. It consists in applying Taylor's expansion in such a way as to require a minimum of numerical calculation. The method is useful in the calculus of residues, since each coefficient is determined independently.

Let the given fraction be $\phi(x)/p(x)$, where $p(x)$ is a polynomial containing

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